01

Units and Measurement

UNITS AND DIMENSIONS

Physical Quantities :

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities

Eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

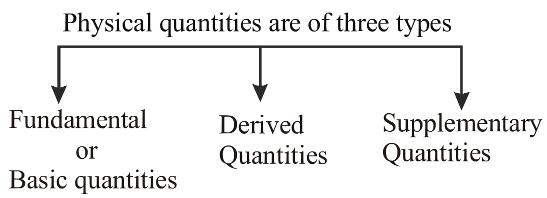
Classification : Physical quantities can be classified on the following bases :

Based on their directional properties

1. **Scalars :** The physical quantities which have only magnitude but no direction are called *scalar quantities*.
   e.g. mass, density, volume, time, etc.
2. **Vectors :** The physical quantities which have both magnitude and direction and obey laws of vector alzebra are called *vector quantities*.
   e.g. displacement, force, velocity, etc.

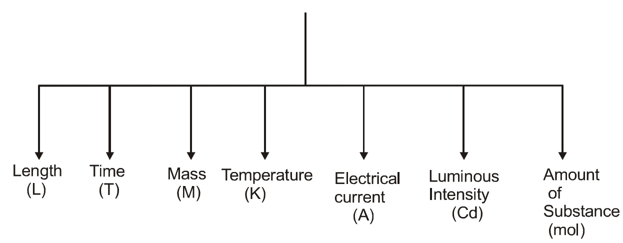
Based on their dependency

1. **Fundamental or base quantities**
2. **Derived quantities**
3. **Supplementary quantities.**



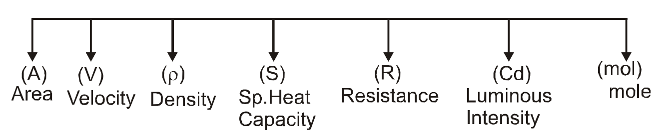
1. Fundamental (Basic) Quantities :

- These are the elementary quantities which covers the entire span of physics.
- Any other quantities can be derived from these.
- All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance (d) , time (t) and velocity (v) cannot be chosen as basic quantities (because they are related as $V = \frac{d}{t}$). An International Organization named CGPM: (Generally known as S.I.)
  General Conference on weight and Measures, chose seven physical quantities as basic or fundamental.



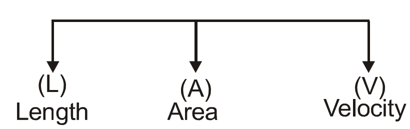
These are the elementary quantities (in our planet) that's why we chosen as basic quantities.

In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived. i.e.,



Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)

But



cannot be used as basic quantities as

Area = $(\text{Length})^2$ so they are not independent.

2. Derived Quantities :

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.

i.e., Momentum

$P = mv$

$$= (m)\frac{\text{displacement}}{\text{time}} = \frac{ML}{T} = M^1 L^1 T^{-1}$$

Here [$M^1 L^1 T^{-1}$] is called dimensional formula of momentum , and we can say that momentum has

1 Dimension in M (mass)

1 Dimension in L (length)

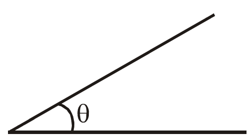
and –1 Dimension in T (time)

The representation of any quantity in terms of basic quantities (M,L,T....) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

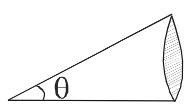
3. Supplementary quantities :

Besides seven fundamental quantities two supplementary quantities are also defined. They are

- Plane angle (The angle between two lines)



- Solid angle subtended by area at a point.



Units of Physical Quantities

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity.

System of Units :

1. **FPS or British Engineering system :** In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.
2. **CGS or Gaussian system :** In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).
3. **MKS system :** In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (m), kilogram (kg) and second (s) respectively.
4. **International system (SI) of units :** This system is modification over the MKS system and so it is also known as *Rationalised MKS system*. Besides the three base units of MKS system four fundamental and two supplementary units are also included in this system.

SI BASE QUANTITIES AND THEIR UNITS

S. No.	Physical quantity	Unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

Note :

While defining a base unit or standard for a physical quantity the following characteristics must be considered :

(i) Well defined

(ii)Invariability (constancy)

(iii) Accessibility (easy availability)

(iv) Reproducibility

(v) Convenience in use

Classification of Units : The units of physical quantities can be classified as follows :

1. **Fundamental or base units :**
The units of fundamental quantities are called *base units*. In SI there are seven base units.
2. **Derived units :**
The units of derived quantities or the units that can be expressed in terms of the base units are called *derived units*.

e.g. unit of speed $= \frac{\text{unit of distance}}{\text{unit of time}} = \frac{\text{metre}}{\text{second}} = ms^{-1}$

Some derived units are named in honour of great scientists.

e.g. unit of force - newton (N),

unit of frequency - hertz (Hz), etc.

3. **Supplementary units :**
In SI two *supplementary units* are also defined viz. radian (rad) for plane angle and steradian (sr) for solid angle.

(i) radian : 1 radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

(ii) steradian : 1 steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere which is equal in area to the square of the radius of the sphere.

Dimensions

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

1. **Dimensional formula :** The *dimensional formula* of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

It is written by enclosing the symbols for base quantities with appropriate powers in square brackets i.e. [ ] e. g. Dim. formula of mass is $[M^1L^0T^0]$ and that of speed (= distance/time) is $[M^0L^1T^{-1}]$

2. **Dimensional equation :** The equation obtained by equating a physical quantity with its dimensional formula is called a *dimensional equation*.
   e.g. $[v] = [M^0L^1T^{-1}]$
   For example $[F] = [MLT^{-2}]$ is a dimensional equation, $[MLT^{-2}]$ is the dimensional formula of the force and the dimensions of force are 1 in mass, 1 in length and –2 in time

Finding Dimensions :

- Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is [L]

Height	L
Width	L
Radius	L
Displacement	L

here [Height] can be read as "Dimension of Height"

- Area = Length × Width
  So, dimension of area is [Area] = [Length] × [Width]
  $= [L] \times [L] = [L^2]$
  For circle
  Area = πr^2
  $[Area] = [\pi][r^2]$
  $= [1][L^2] = [L^2]$
  Here π is not a kind of length or mass or time so π shouldn't affect the dimension of Area.
  Hence its dimension should be 1 $(M^0L^0T^0)$ and we can say that it is dimensionless. From similar logic we can say that all the numbers are dimensionless.

200	$M^0L^0T^0$
– 2	$M^0L^0T^0$
$\frac{1}{4}$	$M^0L^0T^0$

- [Volume] = [Length] × [Width] × [Height]
  $= L \times L \times L = [L^3]$
  For sphere
  $$\text{Volume} = \frac{4}{3}\pi r^3$$
  $$[\text{Volume}] = \left[\frac{4}{3}\pi\right][r^3] = [L^3]$$
  So dimension of volume will be always $[L^3]$ whether it is volume of a cuboid or volume of sphere.
  Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

- $\text{Density} = \dfrac{\text{mass}}{\text{volume}}$
  $$[\text{Density}] = \frac{[\text{mass}]}{[\text{volume}]} = \frac{M}{L^3} = [M^1L^{-3}]$$

- $\text{Velocity (v)} = \dfrac{\text{displacement}}{\text{time}}$
  $$[v] = \frac{[\text{Displacement}]}{[\text{time}]} = \frac{L}{T} = [M^0L^1T^{-1}]$$

- $\text{Acceleration (a)} = \dfrac{dv}{dt}$
  $$[a] = \frac{dv \rightarrow \text{kind of velocity}}{dt \rightarrow \text{kind of time}} = \frac{LT^{-1}}{T} = LT^{-2}$$

- Momentum (P) = mv
  $[P] = [M][v]$
  $= [M][LT^{-1}]$
  $= [M^1L^1T^{-1}]$

- Force (F) = ma
  $[F] = [m][a]$
  $= [M][LT^{-2}]$
  $= [M^1L^1T^{-2}]$

- Work or Energy = force × displacement
  [Work] = [force] [displacement]
  $= [M^1L^1T^{-2}][L]$
  $= [M^1L^2T^{-2}]$

- $\text{Power} = \dfrac{\text{work}}{\text{time}}$
  $$[\text{Power}] = \frac{[\text{work}]}{[\text{time}]} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$$

- $\text{Pressure} = \dfrac{\text{Force}}{\text{Area}}$
  $$[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$$

1. **Dimensions of angular quantities :**

- Angle (θ)
  $$\text{(Angular displacement) } \theta = \frac{\text{Arc}}{\text{radius}}$$
  $$[\theta] = \frac{[\text{Arc}]}{[\text{radius}]} = \frac{L}{L} = [M^0L^0T^0] \text{ (Dimensionless)}$$

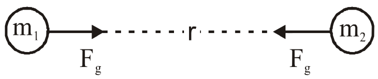
- $\text{Angular velocity } (\omega) = \dfrac{\theta}{t}$
  $$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$$

- $\text{Angular acceleration } (\alpha) = \dfrac{d\omega}{dt}$
  $$[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0L^0T^{-1}}{T} = [M^0L^0T^{-2}]$$

- Torque = Force × Arm length
  [Torque] = [force] × [arm length]
  $= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

2. Dimensions of Physical Constants :

- **Gravitational Constant :**



If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$

where G is a constant called Gravitational constant.

$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$

$[M^1L^1T^{-2}] = \frac{[G][M][M]}{[L^2]}$

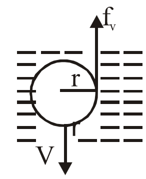
$[G] = M^{-1}L^3T^{-2}$

- **Specific heat capacity :**
  To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$
  Here s is called specific heat capacity.
  $[Q] = [m]\,[s]\,[\Delta T]$
  Here Q is heat : A kind of energy so $[Q] = M^1L^2T^{-2}$
  $[M^1L^2T^{-2}] = [M]\,[s]\,[K]$
  $[s] = [M^0L^2T^{-2}K^{-1}]$

- **Gas constant (R) :**
  For an ideal gas, relation between pressure (P) Value (V), Temperature (T) and moles of gas (n) is
  PV = nRT where R is a constant, called gas constant.
  $[P]\,[V] = [n]\,[R]\,[T]$ .... (1)

here $[P]\,[V] = \frac{[Force]}{[Area]}$ [Area × Length]
= [Force] × [Length]
$= [M^1L^1T^{-2}]\,[L^1] = M^1L^2T^{-2}$
From equation (1)
$[P]\,[V] = [n]\,[R]\,[T]$
$\Rightarrow [M^1L^2T^{-2}] = [mol]\,[R]\,[K]$
$\Rightarrow [R] = [M^1L^2T^{-2}\,mol^{-1}\,K^{-1}]$

- **Coefficient of viscosity :**



If any spherical ball of radius r moves with velocity v in a viscous liquid, then viscous force acting on it is given by
$F_v = 6\pi\eta rv$
Here η is coefficient of viscosity
$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$
$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$
$[\eta] = M^1L^{-1}T^{-1}$

- **Planck's constant :**
  If light of frequency υ is falling , energy of a photon is given by
  $E = h\upsilon$
  Here h = Planck's constant
  $[E] = [h]\,[\upsilon]$

$\upsilon = \text{frequency} = \frac{1}{\text{Time Period}}$

$\Rightarrow [\upsilon] = \frac{1}{[\text{Time Period}]} = \left[\frac{1}{T}\right]$

so $M^1L^2T^{-2} = [h]\,[T^{-1}]$
$[h] = M^1L^2T^{-1}$

Different quantities with units. symbol and dimensional formula.

Quantity	**Symbol**	**Formula**	**S.I. Unit**	**D.F.**
Displacement	s	ℓ	Metre or m	M^0LT^0
Area	A	$\ell \times b$	$(Metre)^2$ or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	$(Metre)^3$ or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	-	F × t	N.sec	MLT^{-1}
Work	W	F . d	N . m	ML^2T^{-2}
Energy	KE or U	$K.E. = \frac{1}{2}mv^2$ P.E. = mgh	Joule or J	ML^2T^{-2}

Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d = mass/volume	kg/m^3	$ML^{-3}T^0$
Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	ML^2T^{-2}
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$M^0L^0T^0$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^0L^0T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^0L^0T^{-2}$
Moment of Inertia	I	$I = mr^2$	$kg\text{-}m^2$	ML^2T^0
Frequency	v or f	$f = \frac{1}{T}$	hertz or Hz	$M^0L^0T^{-1}$
Stress	-	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	-	$\frac{\Delta\ell}{\ell};\frac{\Delta A}{A};\frac{\Delta V}{V}$	-	$M^0L^0T^0$
Youngs modulus	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m^2	$ML^{-1}T^{-2}$
Bulk modulus of rigidity		$\beta = \frac{F/A}{-\Delta v/v}$	N/m^2	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m};\frac{J}{m^2}$	ML^0T^{-2}
Force constant (spring)	k	F = kx	N/m	ML^0T^{-2}
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S.)	$ML^{-1}T^{-1}$
Gravitation constant	G	$F = \frac{Gm_1 m_2}{r^2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^3T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^0L^2T^{-2}$
Temperature	θ	-	Kelvin or K	$M^0L^0T^0\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	ML^2T^{-2}
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{\text{Joule}}{\text{kg.Kelvin}}$	$M^0L^2T^{-2}\theta^{-1}$
Latent heat	L	Q = mL	$\frac{\text{Joule}}{\text{kg}}$	$M^0L^2T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$	$\frac{\text{Joule}}{\text{m sec K}}$	$MLT^{-3}\theta^{-1}$
Universal gas constant	R	PV = nRT	$\frac{\text{Joule}}{\text{mol.K}}$	$ML^2T^{-2}\theta^{-1}$

Quantity	Symbol	Formula	S.I. Unit	D.F.
Mechanical equivalent of heat	J	W = JH	-	$M^0L^0T^0$
Charge	Q or q	$I = \frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	-	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$\varepsilon_0 = \frac{1}{4\pi F}.\frac{q_1q_2}{r^2}$	$\frac{(\text{coul.})^2}{N.m^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}T^4A^2$
Electric potential	V	$V = \frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric	E	$E = \frac{F}{q}$	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	Q = CV	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	-	$M^0L^0T^0$
Resistance	R	V = IR	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^2$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{\ell}$	Ohm × meter	$ML^3T^{-3}A^{-2}$
Conductivity or specific conductance	s	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^3A^2$
Magnetic induction	B	$F = qvB\sin\theta$ or $F = BIL$	Tesla or weber/m^2	$MT^{-2}A^{-1}$
Magnetic flux	φ	$e = \frac{d\phi}{dt}$	Weber	$ML^2T^{-2}A^{-1}$
Magnetic intensity	H	$B = \mu H$	A/m	$M^0L^{-1}T^0A$
Magnetic permeability of free space or medium	μ_0	$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L.\frac{dI}{dt}$	Henery	$ML^2T^{-2}A^{-2}$
Electric dipole moment	p	$p = q \times 2\ell$	C.m.	M^0LTA
Magnetic dipole moment	M	M = NIA	amp.m^2	$M^0L^2AT^0$

DPP-1

Q.1 A standard unit should be
(1) Accessible
(2) Invariable
(3) Internationally accepted
(4) All of these

Q.2 Which of the following does not have dimen sions of force?
(1) Weight
(2) Rate of change of momentum
(3) Work per unit length
(4) Work done per unit charge

Q.3 The term $\frac{1}{2}\rho v^2$ occurs in Bernoulli's equation, with ρ being the density of a fluid and ν its speed. The dimensions of this term are
(1) $M^{-1}L^5T^2$ (2) MLT^2
(3) $ML^{-1}T^{-2}$ (4) $M^{-1}L^9T^{-2}$

Q.4 If C and R denote capacitance and resistance, the dimensional formula of CR is
(1) $[M^0L^0T^1]$
(2) $[M^0L^0T^0]$
(3) $[M^0L^0T^{-1}]$
(4) not expressible in terms of MLT.

Q.5 Find out dimension of $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{hc}$ where e : electronic charge, ε_0 = permittivity of free space, h : planck constant, c : speed of light
(1) $M^1L^1T^{-2}C^2$
(2) $M^2L^2T^{-3}C^2$
(3) $M^1L^1T^{-2}C^2$
(4) Dimension less

Application of Dimensions

1. Principle of Homogenity

This principle states that the dimensions of all the terms in a physical expression should be same. For example, in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of s, ut and $\frac{1}{2}at^2$ all are same.

Note :
The physical quantities separated by the symbols +, –, =, >, < etc., have the same dimensions.

SOLVED EXAMPLE

EXAMPLE 1

The velocity v of a particle depends upon the time t according to the equation $v = a + bt + \frac{c}{d+t}$. Write the dimensions of a, b, c and d.

Sol. From principle of homogeneity

$[a] = [v]$

or $[a] = [LT^{-1}]$

Ans.

$[bt] = [v]$

or $[b] = \frac{[v]}{[t]} = \frac{[LT^{-1}]}{[T]}$

or $[b] = [LT^{-2}]$

Similarly, $[d] = [t] = [T]$ **Ans.**

Further, $\frac{[c]}{[d+t]} = [v]$

or $[c] = [v]\,[d+t]$

or $[c] = [LT^{-1}]\,[T]$

or $[c] = [L]$ **Ans.**

EXAMPLE 2

$$\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$$

Find dimensional formula for [α] and [β] (here t = time, F = force, v = velocity, x = distance)

Sol. Since dimension of Fv = $[Fv] = [M^1L^1T^{-2}]\;[L^1T^{-1}] = [M^1L^2T^{-3}]$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$\frac{[\beta]}{[x^2]} = M^1L^2T^{-3}$

$[\beta] = M^1L^4T^{-3}$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$, so

L.H.S. should also have the same dimension $M^1L^2T^{-3}$

so $\frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$

$[\alpha] = M^1L^2T^{-1}$

EXAMPLE 3

For n moles of gas, Vander waal's equation is

$$\left(P - \frac{a}{V^2}\right)(V - b) = nRT$$

Find the dimensions of a and b, where P is gas pressure, V = volume of gas T = temperature of gas

Sol. $\left(P - \frac{a}{V^2}\right)$ — $\frac{a}{V^2}$ should be a kind of pressure

$(V - b) = nRT$ — b should be a kind of volume

So $\frac{[a]}{[V^2]} = M^1L^{-1}T^{-2}$ So $[b] = L^3$

$\frac{[a]}{[L^3]^2} = M^{-1}L^{-1}T^{-2} \Rightarrow [a] = M^1L^5T^{-2}$

Note :
Consider a term sinθ

Here θ is dimensionless and sinθ $\left(\frac{\text{Perpendicular}}{\text{Hypoteneous}}\right)$

is also dimensionless.

⇒ Whatever comes in sin(......) is dimensionless and entire [sin (.......)] is also dimensionless.

⇒ Dimensions ← sin (....) → Angle dimension less

Similarly :

⇒ Dimensions ← cos (....) → Angle dimensionless

⇒ Dimensions ← tan (....) → Angle dimensionless

⇒ Dimensionless ← $2^{(....)}$ → (....) is dimensionless

⇒ Dimensionless ← $e^{(....)}$ → (....) is dimensionless

⇒ Dimensionless ← $\log_e$(...) → (...) is dimensionless

EXAMPLE 4

$\alpha = \frac{F}{v^2}\sin(\beta t)$ (here v = velocity, F = force, t = time)

Find the dimension of α and β

Sol. $\alpha = \frac{F}{v^2}\sin(\beta t)$

⇒ Dimensionless ← sin (βt) → dimensionless

$\Rightarrow [\beta][t] = 1$

$\Rightarrow [\beta] = [T^{-1}]$

So $[\alpha] = \frac{[F]}{[v^2]} = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

EXAMPLE 5

$\alpha = \frac{Fv^2}{\beta^2}\log_e\left(\frac{2\pi\beta}{v^2}\right)$ where F = force , v = velocity

Find the dimensions of α and β.

Sol. $\alpha = \frac{Fv^2}{\beta^2}\log_e\frac{2\pi\beta}{v^2}$

⇒ Dimensionless ← $\log_e\frac{2\pi\beta}{v^2}$ → dimensionless

$\Rightarrow \frac{[2\pi][\beta]}{[v^2]} = 1$

$\Rightarrow \frac{[1][\beta]}{L^2T^{-2}} = 1$

$\Rightarrow [\beta] = L^2T^{-2}$

as $[\alpha] = \frac{[F][v^2]}{[\beta^2]}$

$\Rightarrow [\alpha] = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2}$

$\Rightarrow [\alpha] = M^1L^{-1}T^0$

2. To check the correctness of the formula :

If the dimensions of the L.H.S and R.H.S are same, then we can say that this eqn. is at least dimensionally correct. So this equation may be correct.

But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct. So it cannot be correct.

e.g. A formula is given centrifugal force $F_e = \frac{mv^2}{r}$

(where m = mass , v = velocity , r = radius)
we have to check whether it is correct or not.
Dimension of L.H.S is
$[F] = [M^1L^1T^{-2}]$
Dimension of R.H.S is

$\frac{[m][v^2]}{[r]} = \frac{[M][LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$

So this eqn. is at least dimensionally correct.
thus we can say that this equation may be correct.

EXAMPLE 6

Check whether this equation may be correct or not.

Pressure $P_r = \dfrac{3Fv^2}{\pi^2 t^2 x}$

(where P_r = Pressure , F = force ,

v = velocity , t = time , x = distance)

Sol. Dimension of L.H.S = $[P_r] = M^1L^{-1}T^{-2}$

Dimension of R.H.S $= \dfrac{[3]\,[F]\,[v^2]}{[\pi]\,[t^2]\,[x]} = \dfrac{[M^1L^1T^{-2}]\,[L^2T^{-2}]}{[T^2]\,[L]}$

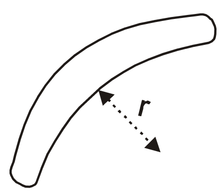
$= M^1L^2T^{-6}$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

Note: Sometimes a question is asked which is beyond our syllabus, then certainly it must be the question of dimensional analyses.

EXAMPLE 7

A Boomerang has mass m surface Area A, radius of curvature of lower surface = r and it is moving with velocity v in air of density ρ. The resistive force on it should be –



(A) $\dfrac{2\rho vA}{r^2}\log\left(\dfrac{\rho m}{\pi Ar}\right)$ (B) $\dfrac{2\rho v^2A}{r}\log\left(\dfrac{\rho A}{\pi m}\right)$

(C) $2\rho v^2A\,\log\left(\dfrac{\rho Ar}{\pi m}\right)$ (D) $\dfrac{2\rho v^2A}{r^2}\log\left(\dfrac{\rho Ar}{\pi m}\right)$

Ans. **(C)**

Sol. Only C is dimensionally correct.

3. To establish the relation among various physical quantities :

If we know the factors on which a given physical quantity may depend, we can find a formula relating the quantity with those factors. Let us take an example.

EXAMPLE 8

The frequency (f) of a stretched string depends upon the tension F (dimensions of force), length l of the string and the mass per unit length μ of string. Derive the formula for frequency.

Sol. Suppose, that the frequency f depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c. Then.

$f \propto [F]^a\,[l]^b\,[\mu]^c$

or $f = k[F]^a\,[l]^b\,[\mu]^c$ ... (i)

Here, k is a dimensionless constant. Thus,

$[f] = [F]^a\,[l]^b\,[\mu]^c$

or $[M^0\,L^0\,T^{-1}] = [MLT^{-2}]^a\,[L]^b\,[ML^{-1}]^c$

or $[M^0L^0T^{-1}] = [M^{a+c}\,L^{a+b-c}\,T^{-2a}]$

For dimensional balance, the dimension on both sides should be same.

Thus,

$a + c = 0$ ...(ii)

$a + b - c = 0$ ...(iii)

and $-2a = -1$ ...(iv)

Solving these three equations, we get

$a = \dfrac{1}{2}$, $c = -\dfrac{1}{2}$ and $b = -1$

Substituting these values in Eq. (i), we get

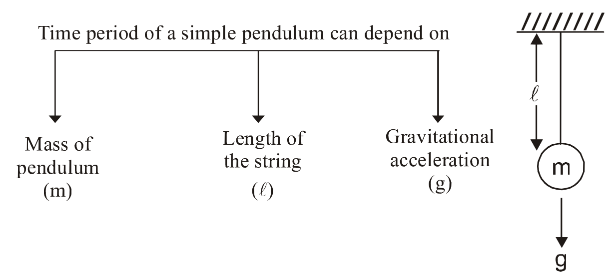
$f = k(F)^{1/2}(l)^{-1}(\mu)^{-1/2}$

or $f = \dfrac{k}{l}\sqrt{\dfrac{F}{\mu}}$

Experimentally, the value of k is found to be $\dfrac{1}{2}$

Hence, $f = \dfrac{1}{2l}\sqrt{\dfrac{F}{\mu}}$

EXAMPLE 9



Sol. So we can say that expression of T should be in this form

$T = (\text{Some Number})\,(m)^a\,(\ell)^b(g)^c$

Equating the dimensions of LHS and RHS,

$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$

$M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$

Comparing the powers of M,L and T,

get $a = 0$, $b + c = 0$, $-2c = 1$

so $a = 0$, $b = \dfrac{1}{2}$, $c = -\dfrac{1}{2}$

so $T = (\text{some Number})\,M^0\,L^{1/2}\,g^{-1/2}$

$T = (\text{Some Number})\,\sqrt{\dfrac{\ell}{g}}$

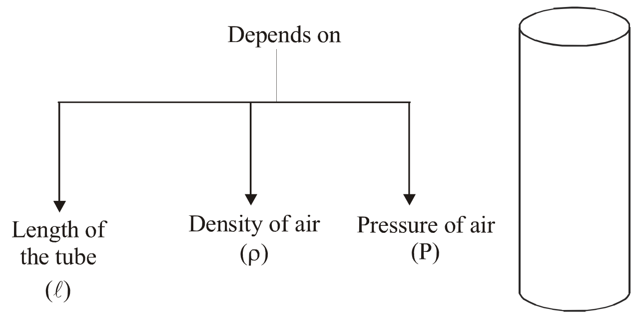
The quantity "Some number" can be found experimentally. Measure the length of a pendulum and oscillate it, find its time period by stopwatch. Suppose for ℓ = 1m, we get T = 2 sec. so

$$2 = (\text{Some Number})\sqrt{\frac{1}{9.8}}$$

$\Rightarrow$ "Some number" = 6.28 $\approx 2\pi$.

EXAMPLE 10

Natural frequency (f) of a closed pipe



Sol. So we can say that

f = (some Number) $(\ell)^a (\rho)^b (P)^c$

$$\left[\frac{1}{T}\right] = (1)\,[L]^a\,[ML^{-3}]^b\,[M^1L^{-1}T^{-2}]^c$$

$$M^oL^oT^{-1} = M^{b+c}\,L^{a-3b-c}\,T^{-2c}$$

comparing powers of M, L, T

$0 = b + c$

$0 = a - 3b - c$

$-1 = -2c$

get $a = -1$, $b = -1/2$, $c = 1/2$

So $f = (\text{some number})\,\frac{1}{\ell}\sqrt{\frac{P}{\rho}}$

Conversion of Units :

This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, i.e.,

$n[u]$ = constant

or $n_1[u_1] = n_2[u_2]$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then we can write.

$$n_1[M_1^a\,L_1^b\,T_1^c] = n_2[M_2^a\,L_2^b\,T_2^c] \quad ...(i)$$

Here n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

EXAMPLE 11

The value of gravitation constant is G = 6.67 × 10^{-11} Nm^2/kg^2 in SI units. Convert it into CGS system of units.

Sol. The dimensional formula of G is $[M^{-1}\,L^3\,T^{-2}]$.

Using equation number (i), i.e.,

$$n_1[M_1^{-1}\,L_1^3\,T_1^{-2}] = n_2[M_2^{-1}\,L_2^3\,T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^3\left[\frac{T_1}{T_2}\right]^{-2}$$

Here, $n_1 = 6.67 \times 10^{-11}$

M_1 = 1 kg, $M_2 = 1\text{ g} = 10^{-3}$ kg

L_1 = 1 m, $L_2 = 1\text{cm} = 10^{-2}$ m,

$T_1 = T_2 = 1$s

Substituting in the above equation, we get

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\,\text{kg}}{10^{-3}\,\text{kg}}\right]^{-1}\left[\frac{1\,\text{m}}{10^{-2}\,\text{m}}\right]^3\left[\frac{1\,\text{s}}{1\,\text{s}}\right]^{-2}$$

or $n_2 = 6.67 \times 10^{-8}$

Thus, value of G in CGS system of units is 6.67 × 10^{-8} dyne cm^2/g^2.

Limitations of Dimensional Analysis

The method of dimensions has the following limitations:

(i) By this method the value of dimensionless constant can not be calculated.

(ii) By this method the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.

(iii) If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T.

Estimates and Order of Magnitude Calculations

It is often useful to compute an approximate answer to a physical problem even where little information is available. Such an approximate answer can then be used to determine whether a more accurate calculation is necessary. Approximations are usually based on certain assumptions, which must be modified if greater accuracy is needed. Thus, we shall sometimes refer to the order of magnitude of a certain quantity as the power of ten of the number that describes that quantity.

In this method, each number is expressed as a × b^b where $1 \le a < 10$ and b is a positive or negative integer. Thus the diameter of the sun is expressed as 1.39×10^9 m and the diameter of a hydrogen atom as 1.06×10^{-10} m. To get an approximate idea of the number, one may round the number a to 1 if it is less than or equal to 5 and to 10 if it is greater than 5. The number can then be expressed approximately as 10^b. We then get the order of magnitude of that number thus, the diameter of the sun is of the order of 10^9 m and that of a hydrogen atom is of the order of 10^{-10} m. More precisely, the exponent of 10 in such a representation is called the order of magnitude of that quantity. Thus, the diameter of the sun is 19 orders of magnitude larger than the diameter of a hydrogen atom. This is because the order of magnitude of 10^9 is 9 and 10^{-10} is – 10. The difference is 9 – (–10) = 19.

The spirit of order of magnitude calculations, sometimes referred to as "guesstimates" or "ball-park figures," is given in the following quotation: " Make an estimate before every calculations try a simple physical argument, before every derivation, guess the answer to every puzzle. No one else needs to know what the guess is. " Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates get better and better. Estimations problems can be fun to work as you freely drop digits. venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head.

SOLVED EXAMPLE

EXAMPLE 12

Order of magnitude of the following values can be determined as follows :

(a) $49 = 4.9 \times 10^1 \approx 10^1$

$\therefore$ Order of magnitude = 1

(b) $51 = 5.1 \times 10^1 \approx 10^2$

$\therefore$ Order of magnitude = 2

(c) $0.049 = 4.9 \times 10^{-2} \approx 10^{-2}$

$\therefore$ Order of magnitude = –2

(d) $0.050 = 5.0 \times 10^{-2} \approx 10^{-1}$

$\therefore$ Order of magnitude = –1

(e) $0.051 = 5.1 \times 10^{-2} \approx 10^{-1}$

$\therefore$ Order of magnitude = –1

Self Practice Problems

Q.1 Give the order of the following :

(a) 1
(b) 1000
(c) 499
(d) 500
(e) 501
(f) 1 AU (1.496×10^{11} m)
(g) 1 Å (10^{-10} m)
(h) Speed of light (3.00×10^8 m/s)
(i) Gravitational constant (6.67×10^{-11} N – m^2/kg^2)
(j) Avogadro constant (6.02×10^{23} mol^{-1})
(k) Planck's constant (6.63×10^{-34} J-s)
(l) Charge on electron (1.60×10^{-19} C)
(m) Radius of H– atom (5.29×10^{-11} m)
(n) Atmospheric pressure (1.01×10^5 Pa)
(o) Mass of earth (5.98×10^{24} kg)
(p) Mean radius of earth (6.37×10^6 m)

Ans.

(a) 0	(b) 3	(c) 2	(d) 3
(e) 3	(f) 11	(g) –10	(h) 8
(i) –10	(j) 24	(k) –33	(l) –19
(m) –10	(n) 5	(o) 25	(p) 7

Breaths in Lifetime

Estimate the number of breaths taken during an average life span.

Sol. We shall start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so fourth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is approximately.

$$1\text{ yr} \times 400\,\frac{\text{days}}{\text{yr}} \times 25\,\frac{\text{h}}{\text{day}} \times 60\,\frac{\text{min}}{\text{h}} = 6 \times 10^5\text{ min}$$

Notice how much simpler it is to multiply 400 × 25 than it is to work with the more accurate 365 × 24. These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 yr. there will be (70 yr) (6×10^5 min/yr) - 4×10^7 min. At a rate of 10 breaths/ min, an individual would take 4×10^8 breaths in a lifetime.

Basic Mathematics

Mensuration Formulas :

r : radius ; d = diameter ; V = Volume; S.A = surface area

(a) Circle

Perameter : $2\pi r = \pi d$, Area : $\pi r^2 = \frac{1}{4}\pi d^2$

(b) Sphere

Surface area = $4\pi r^2 = \pi d^2$, Volume = $\frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$

(c) Spherical Shell (Hollow sphere)

Surface area = $4\pi r^2 = \pi d^2$

Volume of material used = $(4\pi r^2)(dr)$,

dr = thickness

(d) Cylinder

Lateral area = $2\pi rh$

$V = \pi r^2 h$

Total area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

(e) Cone

Lateral area $= \pi r \sqrt{r^2 + h^2}$

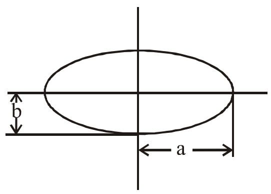
h = height

Total area $= \pi r \left(\sqrt{r^2 + h^2} + r\right)$

$$V = \frac{1}{3}\pi r^2 h$$

(f) Ellipse

Circumference $\approx 2\pi\sqrt{\frac{a^2 + b^2}{2}}$



area $= \pi ab$

a = semi major axis

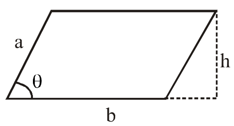
b = semi minor axis

(g) Parallelogram

A = bh = ab sin θ

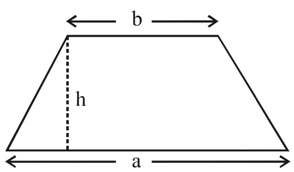
a = side ; h = height ; b = base

θ=angle between sides a and b



(h) Trapezoid

$$\text{area} = \frac{h}{2}(a + b)$$



a and b parallel sides

h = height

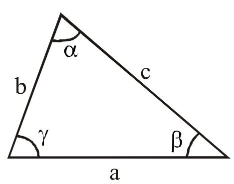
(i) Triangle

$$\text{area} = \frac{bh}{2} = \frac{ab}{2}\sin\gamma\sqrt{s(s-a)(s-b)(s-c)}$$

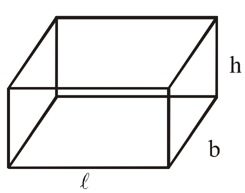
a, b, c sides are opposite to angles α, β, γ

b = base ; h = height

$$s = \frac{1}{2}(a+b+c)$$



(j) Rectangular container



side ℓ,b,h

lateral area $= 2(\ell b + bh + h\ell)$; $V = \ell bh$

Mathematics is the language of physics. It becomes easier to describe, understand and apply the physical principles, if one has a good knowledge of mathematics.

Logarithms :

(i) e ≈ 2.7183

(ii) If $e^x = y$, then $x = \log_e y = \ln y$

(iii) If $10^x = y$, then $x = \log_{10} y$

(iv) $\log_{10} y = 0.4343 \log_e y = 2.303 \log_{10} y$

(v) log (ab) = log (a) + log (b)

(vi) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

(vii) $\log a^n = n \log(a)$

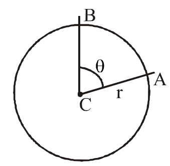
Trigonometric Properties :

(i) Measurement of angle & relationship between degrees & radian

In navigation and astronomy, angles are measured in degrees, but in calculus it is best to use units called radians because of they simplify later calculations.

Let ACB be a central angle in circle of radius r, as in figure.

Then the angle ACB or θ is defined in radius as -



$$\theta = \frac{\text{Arc length}}{\text{Radius}} \Rightarrow \theta = \frac{\widehat{AR}}{r}$$

If r = 1 then θ = AB

The radian measure for a circle of unit radius of angle ABC is defined to be the length of the circular arc AB. since the circumference of the cirlce is 2π and one complete revolution of a cicle is 360°, the relation between radians and degrees is given by the following equation.

π radians = 180°

(ii) ANGLE CONVERSION FORMULAS

$$1 \text{ degree} = \frac{\pi}{180°} \quad (\approx 0.02) \text{ radian}$$

Degrees to radians : multiply by $\frac{\pi}{180°}$

1 radian ≈ 57 degrees

Radians to degrees : multiply by $\frac{180°}{\pi}$

SOLVED EXAMPLE

EXAMPLE 13

(a) Covert 45° to radians.

(b) Convert $\frac{\pi}{6}$ rad to degrees.

Sol. (a) $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ rad

(b) $\frac{\pi}{6} \cdot \frac{180}{\pi} = 30°$

EXAMPLE 14

Convert 30º to radians :

Sol. $30^o \times \frac{\pi}{180^o} = \frac{\pi}{6}$ rad

EXAMPLE 15

Convert $\frac{\pi}{3}$ rad to degrees.

Sol. $\frac{\pi}{3}\times\frac{180}{\pi}=60$

Standard values

(1) $30^\circ = \frac{\pi}{6}$rad (2) $45^\circ = \frac{\pi}{4}$rad

(3) $60^\circ = \frac{\pi}{3}$rad (4) $90^\circ = \frac{\pi}{2}$rad

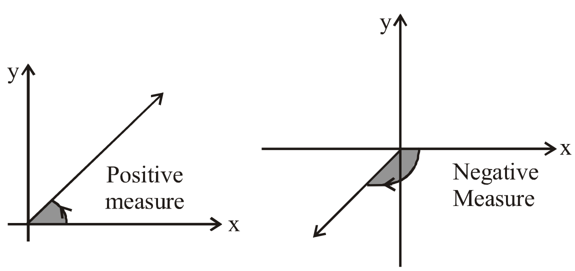
(5) $120^\circ = \frac{2\pi}{3}$rad (6) $135^\circ = \frac{3\pi}{4}$rad

(7) $150^\circ = \frac{5\pi}{6}$rad (8) $180^\circ = \pi$ rad

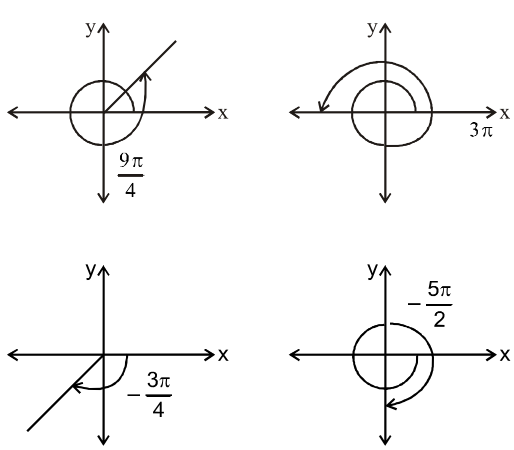
(9) $360^\circ = 2\pi$ rad

(Check these values yourself to see that the satisfy the conversion formulaes)

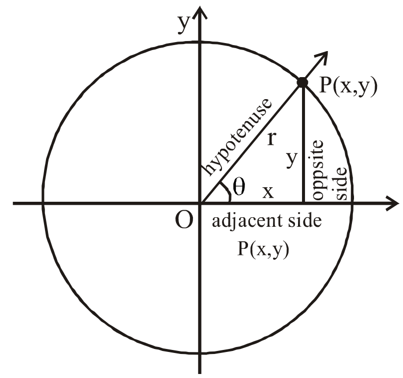
(ii) Measurement of positive & Negative Angles:



An angle in the xy-plane is said to be in standard position if its vertex lies at the origin and its initial ray lies along the positive x-axis (Fig). Angles measured counterclockwise from the positive x-axis are assigned positive measures; angles measured clockwise are assigned negative measures.



(iii) Six Basic Trigonometric Functions :



The trigonometric fucntion of a general angle θ are defined in tems of x, y and r.

Sine : $\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

Cosecant : $\text{cosec}\,\theta = \frac{hyp}{opp} = \frac{r}{y}$

Cosine: $\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$

Secant : $\sec\theta = \frac{hyp}{adj} = \frac{r}{x}$

Tangent: $\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

Cotangent: $\cot\theta = \frac{adj}{opp} = \frac{x}{y}$

VALUES OF TRIGONOMETRIC FUNCTIONS

If the circle in (Fig. above) has radius r = 1, the equations defining sin θ and cos θ become

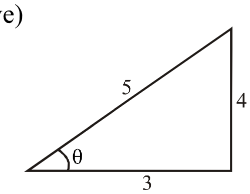
$\cos\theta = x$, $\sin\theta = y$

We can then calculate the values of the cosine and sine directly from the coordinates of P.

SOLVED EXAMPLE

EXAMPLE 16

Find the six trigonometric ratios from given fig. (see above)



Sol. $\sin\theta = \frac{opp}{hyp} = \frac{4}{5}$ $\cos\theta = \frac{adj}{hyp} = \frac{3}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$ $\qquad$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$ $\qquad$ $\text{cosec}\,\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$

EXAMPLE 17

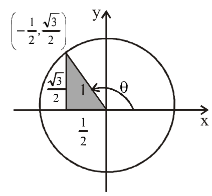
Find the sine and cosine of angle θ shown in the unit circle if coordinate of point p are as shown.

Sol. cos θ = x-coordinate of

$P = -\frac{1}{2}$

sin θ = y-coordinate of

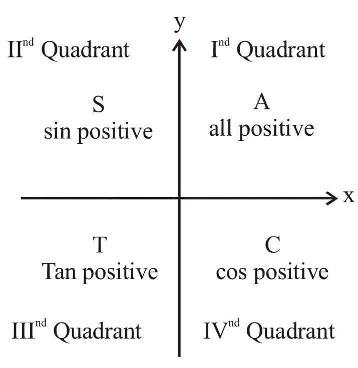
$P = \frac{\sqrt{3}}{2}$



Values of sin θ, cos θ and tan θ for some standard angles.

Degree	0	30	37	45	53	60	90	120	135	180
Radians	0	$\pi/6$	$37\pi/180$	$\pi/4$	$53\pi/180$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$\sin\theta$	0	1/2	3/5	$1/\sqrt{2}$	4/5	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	0
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$1/\sqrt{2}$	3/5	1/2	0	–1/2	$-1/\sqrt{2}$	–1
$\tan\theta$	0	$1/\sqrt{3}$	3/4	1	4/3	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	0

A useful rule for remembering when the basic trigonometric funcions are positive and negative is the CAST rule. If you are not very enthusiastic about CAST. You can remember it as ASTC (After school to college)



The CAST rule

(iii) RULES FOR FINDING TRIGONOMETRIC RATIO OF ANGLES GREATER THAN 90°.

Step 1 → Identify the quadrant in which angle lies.

Step 2 → (a) If angle = $(n\pi \pm \theta)$ where n is an integer. Then put $(n\pi \pm \theta) = \theta$ and sign will be decided by CAST Rule.

(b) If angle $= \left[(2n+1)\frac{\pi}{2} + \theta\right]$ where n is in interger.

Then trigonometric function of $\left[(2n+1)\frac{\pi}{2} \pm \theta\right]$ = complimentry trignometric function of θ and sign will be decided by CAST Rule.

EXAMPLE 18

Evaluate sin 120°

Sol. $\sin 120° = \sin(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

Aliter $\sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

EXAMPLE 19

Evaluate cos 210°

Sol. $\cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

EXAMPLE 20

$\tan 210° = \tan(180° + 30°) = \tan 30° = +\frac{1}{\sqrt{3}}$

Important Formulas

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

(iv) $\sin 2\theta = 2\sin\theta\cos\theta$

(v) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$

(vi) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(vii) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(viii) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ix) $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

(x) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(xi) $\cos C - \cos D = 2\sin\frac{D-C}{2}\sin\frac{C+D}{2}$

(xii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(xiii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

(xiv) $\sin(90° + \theta) = \cos\theta$

(xv) $\cos(90° + \theta) = -\sin\theta$

(xvi) $\tan(90° + \theta) = -\cot\theta$

(xvii) $\sin(90° - \theta) = \cos\theta$

(xviii) $\cos(90° - \theta) = \sin\theta$

(xix) $\cos(180° - \theta) = -\cos\theta$

(xx) $\sin(180° - \theta) = \sin\theta$

(xxi) $\cos(180° + \theta) = -\cos\theta$

(xxii) $\tan(180° + \theta) = \tan\theta$

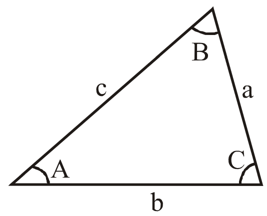
(xxiii) $\sin(-\theta) = -\sin\theta$

(xxiv) $\cos(-\theta) = \cos\theta$

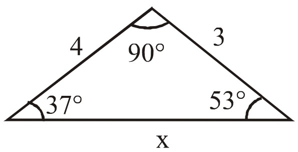
(xxv) $\tan(-\theta) = -\tan\theta$

• **Sine Rule** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

• **Cosine rule** $a^2 = b^2 + c^2 - 2bc \cos A$



EXAMPLE 21



Find x :

Sol. $\frac{\sin 90°}{x} = \frac{\sin 53°}{4}$

$x = 5$

(iv) small angle approximation

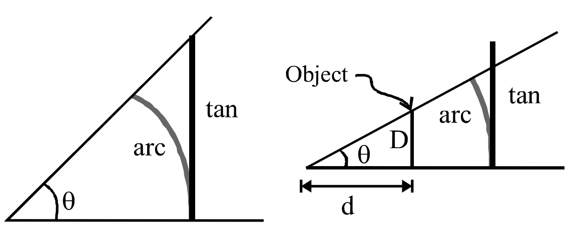
It is a useful simplification which is only approximately true for finite angles. It involves linerarization of the trigonometric functions so that, when the angle θ is measured in radians.

$\sin\theta \simeq \theta$

$\cos\theta \simeq 1$ or $\cos\theta \simeq 1 - \frac{\theta^2}{2}$ for the second - order approximation

$\tan\theta \simeq \theta$

Geometric justification



Small angle approximation. The value of the small angle X in radians is approximately equal to its tangent.

- When one angle of a right triangle is small, is hypotenuse in approximately equal in length to the leg adjacent to the small angle, so the cosine is approximately 1.
- The short leg is approximately equal to the arc from the long leg to the hypotenuse, so the sine and tangent are both approximated by the value of the angle in radians.

Binomial Theorem :

$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!}$..........

$(1 \pm x)^{-n} = 1 \mp nx + \frac{n(n+1)}{2!}x^2$........

If $x << 1$; then

$(1 \pm x)^n = 1 \pm nx$ (neglecting higher terms)

$(1 \pm x)^{-n} = 1 \pm (-n)x = 1 \mp nx$

$(1 + x)^2 = 1 + 2x + x^2$

$(1 + x)^3 = 1 + 3x + x^3 - 3x^2$

$(1 + x)^n = 1 + nx$ ....

if $x << 1$

> **Note :**
> **(1)** When n is a positive integer, then expansion will have (n + 1) terms
> **(2)** When n is a negative integer, expansion will have infinite terms.
> **(3)** When n is a fraction expansion will have infinite terms.

EXAMPLE 22

Calculate $(1001)^{1/3}$.

Sol. We can write 1001 as : $1001 = 1000\left(1 + \frac{1}{1000}\right)$, so that

we have

$$(1001)^{1/3} = \left[1000\left(1 + \frac{1}{1000}\right)\right]^{1/3} = 10\left[1 + \frac{1}{1000}\right]^{1/3}$$

$$= 10(1 + 0.001)^{1/3} = 10(1 + \frac{1}{3} \times 0.001)$$

$= 10.003333$

EXAMPLE 23

Expand $(1+x)^{-3}$.

Sol. $$(1 + x)^{-3} = 1 + (-3)x + \frac{(-3)(-3-1)x^2}{2!} + \frac{(-3)(-3-1)(-3-2)}{3!}x^3 +$$

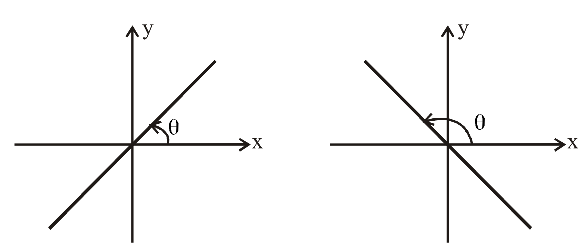
$$= 1 - 3x + \frac{12}{2}x^2 - \frac{60}{3 \times 2}x^3 + ...$$

$$= 1 - 3x + 6x^2 - 10x^3 + ...$$

Graphs :

Following graphs and their corresponding equations are frequently used in Physics.

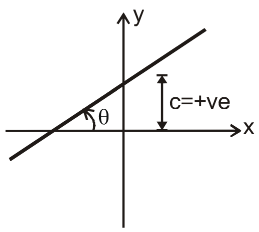
(i) $y = mx$, represents a straight line passing through origin. Here, $m = \tan\theta$ is also called the slope of line, where θ is the angle which the line makes with positive x-axis, when drawn in anticlockwise direction from the positive x-axis towards the line.



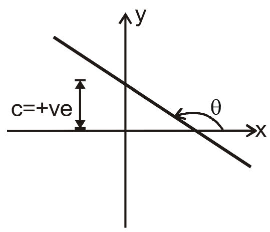
(i) (ii)

The two possible cases are shown in figure 1.1 (i) $\theta < 90°$. Therefore, $\tan\theta$ or slope of line is positive. In fig. 1.1 (ii), $90° < \theta < 180°$. Therefore, $\tan\theta$ or slope of line is negative.

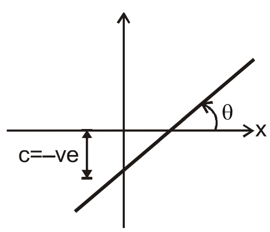
Note : That $y = mx$ of $y \propto x$ also means that value of y becomes 2 time if x is doubled. Or it becomes $\frac{1}{4}$th if x becomes $\frac{x}{4}$, and c the intercept on y-axis.



(i)



(ii)



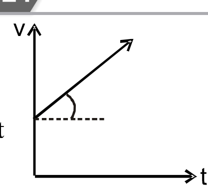
(iii)

In figure (i) : slope and intercept both are positive.
In figure (ii) : slope is negative but intercept is positive and In figure (iii) : slope is positive but intercept is negative.

> **Note :**
>
> That in $y = mx + c$, y does not become two times if x is doubled

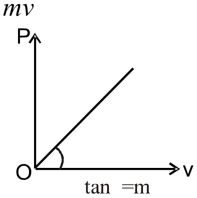
EXAMPLE 24

$v = u + at$



EXAMPLE 25

$P = mv$



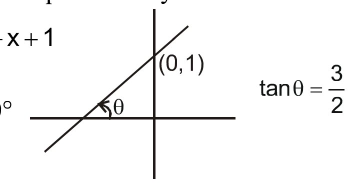
EXAMPLE 26

Draw the graph for the equation : $2y = 3x + 2$

Sol. $2y = 3x + 2 \Rightarrow y = \frac{3}{2}x + 1$

$m = \frac{3}{2} > 0 \Rightarrow \theta < 90°$

$c = +1 > 0$



$\Rightarrow$ The line will pass through (0, 1)

EXAMPLE 27

Draw the graph for the equation : $2y + 4x + 2 = 0$

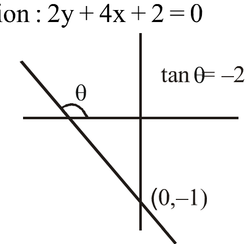
Sol. $2y + 4x + 2 = 0$

$\Rightarrow y = -2x - 1$

$m = -2 < 0$ i.e., $\theta > 90°$

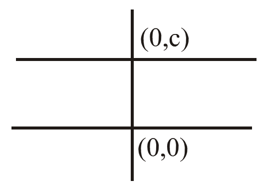
$c = -1$ i.e.,

line will pass through (0, –1)

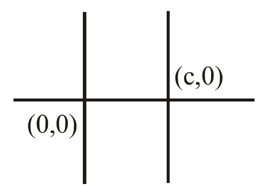


(i) If $c = 0$ line will pass through origin.

(ii) $y = c$ will be a line parallel to x axis.



(iii) $x = c$ will be a line perpendicular to x axis



(ii) Parabola

A general quadratic equation represents a parabola.

$y = ax^2 + bx + c$

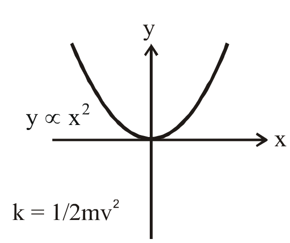
$a \neq 0$

if $a > 0$; It will be a opening upwards parabola.

if $a < 0$; It will be a opening downwards parabola.

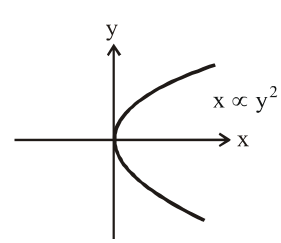
if $c = 0$; It will pass through origin.

$y \propto x^2$ or $y = 2x^2$, etc. represents a parabola passing through origin as shown in figure shown.



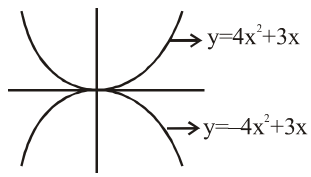
(i)

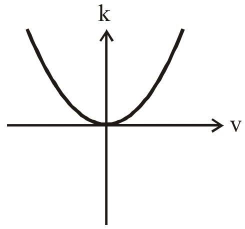
e.g. $y = 4x^2 + 3x$



(ii)

e.g. $k = \frac{1}{2}mv^2$

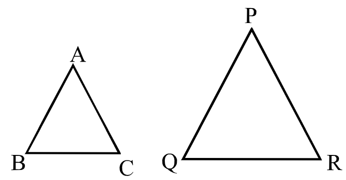




Note :
That in the parabola $y = 2x^2$ or $y \propto x^2$, if x is doubled, y will beome four times.
Graph $x \propto y^2$ or $x = 4y^2$ is again a parabola passing through origin as shown in figure shown. In this cae if y is doubled, x will become four times.
$y = x^2 + 4$ or $x = y^2 - 6$ will represent a parabola but not passing through origin. In the first equation $(y = x^2 + 4)$, if x doubled, y will not become four times.

Similar Triangle

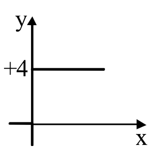
Two given triangle are said to be similar if
(1) All respective angle are same
or
(2) All respective side ratio are same.

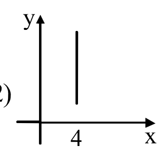


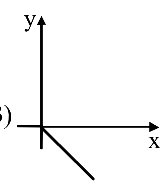
As example, ABC, PQR are two triangle as shown in figure.

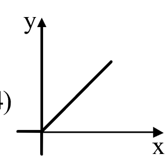
DPP-2

Q.1 Which graph represent $y = 4x$?

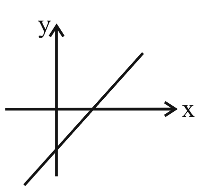
(1) 

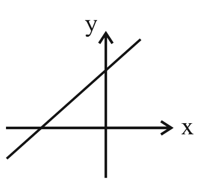
(2) 

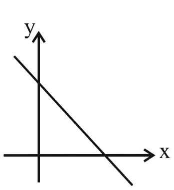
(3) 

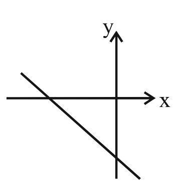
(4) 

Q.2 Which of the following graph is the best representation for the given equation : $y = 2x - 1$

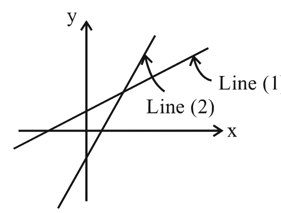
(1) 

(2) 

(3) 

(4) 

Q.3 Which of the following statement is not correct for following straight line graph :



(1) Line (2) has negative y intercept
(2) Line (1) has positive y intercept
(3) Line (2) has positive slope
(4) Line (1) has negative slope

Q.4 The value of $(997)^{1/3}$ according to binomial theorem is
(1) 9.00 (2) 9.99
(3) 10.90 (4) 9.33

Q.5 The value of $K\left[\left(1+\dfrac{\Delta T}{T_0}\right)^4 - 1\right]$ (where $\Delta T << T_0$) according to Binomial theorem is

(1) $\dfrac{K\Delta T}{T_0}$ (2) $\dfrac{2K\Delta T}{T_0}$

(3) $\dfrac{4K\Delta T}{T_0}$ (4) $\dfrac{K\Delta T}{4T_0}$

Q.6 The value of cos (–60°) is

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$

(3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{\sqrt{3}}{2}$

Q.7 tan(A+B) =

(1) $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

(2) $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

(3) $\frac{\tan A + \tan B}{1 + \tan A \tan B}$

(4) $\frac{\tan A \tan B}{\tan A + \tan B}$

Q.8 The approximate value of x where x= sin 2° cos2° is

(1) $\frac{\pi}{90}$ (2) 2

(3) 1 (4) $\frac{\pi}{45}$

Q.9 Which of the following is correct option ?

(1) $\sin^2\theta = 1+\cos^2\theta$

(2) $1- \tan^2\theta = \sec^2\theta$

(3) $1+ \cot^2\theta = \text{cosec}^2\theta$

(4) None of these

Q.10 The value of sin300°

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$

(3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{\sqrt{3}}{2}$

ERROR ANALYSIS IN EXPERIMENTS

Significant Figures or Digits

The *significant figures* (SF) in a measurement are the figures or digits that are known with certainity plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

1. Rules to find out the number of significant figures :

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

IV Rule : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule : The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

VI Rule : The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. e.g. m = 100 kg has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

2. Rules for arithmetical operations with significant figures :

I Rule : In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule : In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $5.0 \times 0.125 = 0.625 = 0.62$

To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in *scientific notation* (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number (a) is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).

The *change in the unit of measurement of a quantity does not affect the number of SF*. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 μm each term has 4 SF.

SOLVED EXAMPLE

EXAMPLE 28

Write down the number of significant figures in the following.

(a) 165	3SF (following rule I)
(b) 2.05	3 SF (following rules I & II)
(c) 34.000 m	5 SF (following rules I & V)
(d) 0.005	1 SF (following rules I & IV)
(e) 0.02340 N m^{-1}	4 SF (following rules I, IV & V)
(f) 26900	3 SF (see rule VI)
(g) 26900 kg	5 SF (see rule VI)

EXAMPLE 29

The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. length (ℓ) = 4.234 m breadth (b) = 1.005 m

thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore area of the sheet = $2(\ell \times b + b \times t + t \times \ell)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ m^2$

$= 2(4.3604739)\ m^2 = 8.720978\ m^2$

Since area can contain a max^m of 3 SF (Rule II of article 4.2) therefore, rounding off, we get

Area = 8.72 m^2

Like wise volume = $\ell \times b \times t$

$= 4.234 \times 1.005 \times 0.0201\ m^3 = 0.0855289\ m^3$

Since volume can contain 3 SF, therefore, rounding off, we get

Volume = 0.0855 m^3

Self Practice Problems

Q.2 Write the following in scientific notation :

(a) 3256 g (b) .0010 g

(c) 50000 g (5 SF) (d) 0.3204

Q.3 Give the number of significant figures in the following:

(a) 0.165 (b) 4.0026

(c) 0.0256 (d) 165

(e) 0.050 (f) 2.653×10^4

(g) 6.02×10^{23} (h) 0.0006032

Q.4 Calculate area enclosed by a circle of diameter 1.06 m to correct number of significant figures.

Q.5 Subtract 2.5×10^4 from 3.9×10^5 and give the answer to correct number of significant figures.

Q.6 The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) total mass of the box (b) the difference in masses of gold pieces to correct significant figures.

Answers

2. (a) 3.256×10^3 g (b) 1.0×10^{-3} g
   (c) 5.0×10^4 g (d) 3.204×10^{-1}
3. (a) 3 (b) 5
   (c) 3 (d) 3
   (e) 2 (f) 4
   (g) 3 (h) 4
4. $0.882\ m^2$ (3 SF)
5. 3.6×10^5
6. (a) Total mass = 2.3 kg
   (b) Difference in masses = 0.02g

Rounding Off

To represent the result of any computation containing more than one uncertain digit, it is *rounded off* to appropriate number of significant figures.

Rules for rounding off the numbers :

I Rule : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

II Rule : If the digit to be rounded off is less than 5, than the preceding digit is unaffected and is left unchanged. e.g. $3.94 \approx 3.9$

III Rule : If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$

SOLVED EXAMPLE

EXAMPLE 30

The following values can be rounded off to four significant figures as follows :

(a) $36.879 \approx 36.88$

($\because$ 9 > 5 $\therefore$ 7 is increased by one i.e.I Rule)

(b) $1.0084 \approx 1.008$

($\because$ 4 < 5 $\therefore$ 8 is left unchanged i.e. II Rule)

(c) $11.115 \approx 11.12$

($\because$ last 1 is odd it is increased by one i.e.III Rule)

(d) $11.1250 \approx 11.12$

($\because$ 2 is even it is left unchanged i.e. III Rule)

(e) $11.1251 \approx 11.13$

($\because$ 51 > 50 $\therefore$ 2 is incresed by one i.e. I Rule)

Self Practice Problems

Q.7 Round off the following numbers as indicated:

(a) 25.653 to 3 digits (b) 4.996×10^5 to 3 digits

(c) 0.6995 to 1digit (d) 3.350 to 2 digits

(e) 0.03927 kg to 3 digits (f) 4.085×10^8 s to 3 digits

Ans. 7. (a) 25.7 (b) 5.00×10^5 (c) 0.7

(d) 3.4 (e) 0.0393 kg (f) 4.08×10^8 s

ERRORS IN MEASUREMENT

Definition

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Classification of Errors

Errors may arise from different sources and are usually classified as follows :-

Systematic or Controllable Errors : Systematic errors are the errors whose causes are known. They can be either positive or negative. Due to the known causes these errors can be minimised. Systematic errors can further be classified into three categories :

(i) **Instrumental errors :-** These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.

(ii) **Environmental errors :-** These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.

(iii) **Observational errors :-** These errors arise due to improper setting of the apparatus or carelessness in taking observations.

Random Errors : These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random erros can be reduced by repeating the observation a large number of times and taking the arithmetic mean of all the obervations. This mean value would be very close to the most accurate reading.

Note :- If the number of observations is made *n* times then the random error reduces to $(\frac{1}{n})$ times.

Example :- If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$

Gross Errors : Gross errors arise due to human carelessness and mistakes in reading the instruments or calculating and recording the measurement results.

For example :-

(i) Reading instrument without proper initial settings.

(ii) Taking the observations wrongly without taking necessary precautions.

(iii) Exhibiting mistakes in recording the observations.

(iv) Putting improper values of the observations in calculations.

These errors can be minimised by increasing the sincerity and alertness of the observer.

Representation of Errors

Errors can be expressed in the following ways :-

1. **Mean Absolute Error :-** It is given by

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}$$

$$a_m = \frac{a_1 + a_2 + ... + a_n}{n}$$ = is taken as the true value of a quantity, if the same is not known.

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

....

$\Delta a_n = a_m - a_n$

Final result of measurement may be written as :

$a = a_m \pm \Delta\bar{a}$

2. **Relative Error or Fractional Error :** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \frac{\overline{\Delta a}}{a_m} \times 100\%$

SOLVED EXAMPLE

EXAMPLE 31

The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. find (i) mean time period (ii) absolute error in each observation and percentage error.

Sol. (i) Meam time period is given by

$$\overline{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5} = \frac{13.12}{5} = 2.62\text{s}$$

(ii) The absolute error in each observation is

$2.62 - 2.63 = -0.01, 2.62 - 2.56 = 0.06, 2.62 - 2.42 = 0.20,$
$2.62 - 2.71 = -0.09, 2.62 - 2.80 = -0.18$

Mean absolute error, $\overline{\Delta T} = \frac{\Sigma|\Delta T|}{5}$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11\text{sec}$$

$$\therefore \text{ Percentage error } = \frac{\overline{\Delta T}}{\overline{T}} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$$

Combination of Errors :

(i) In Sum : If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$, maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A + B} + \frac{\Delta B}{A + B}$$

i.e. when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(ii) In Difference : If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case $\frac{\Delta Z}{Z} = \frac{\Delta A}{A - B} + \frac{\Delta B}{A - B}$

(iii) In Product : If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

(iv) In Division : If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) In Power : If $Z = A^n$ then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

In more general form if $Z = \frac{A^x B^y}{C^q}$

then the maximum fractional error in *Z* is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

Applications :

1. For a simple pendulum, $T \propto l^{1/2}$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

2. For a sphere

$$A = 4\pi r^2, V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{\Delta A}{A} = 2.\frac{\Delta r}{r} \text{ and } \frac{\Delta V}{V} = 3.\frac{\Delta r}{r}$$

3. When two resistors R_1 and R_2 are connected

(a) In series

$$R_s = R_1 + R_2$$

$$\Rightarrow \Delta R_s = \Delta R_1 + \Delta R_2$$

$$\frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

(b) In parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

SOLVED EXAMPLE

EXAMPLE 32

In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of $\frac{L}{T^2}$ is

(1) 5% (2) 7% (3) 8% (4) 1%

Sol. (2) Maximum percentage in the value of $\frac{L}{T^2}$

$$= \frac{\Delta L}{L}\times 100\% + 2\frac{\Delta T}{T}\times 100\%$$

$$= 3 + 2\times 2 = 7\%$$

EXAMPLE 33

If $X = \frac{A^2\sqrt{B}}{C}$, then

(1) $\Delta X = \Delta A + \Delta B + \Delta C$

(2) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

(3) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$

(4) $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

Ans. 3

Sol. $\because X = A^2 B^{1/2} C \quad \therefore \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$

EXAMPLE 34

A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity ?

Sol. Given distance, $s = (13.8 \pm 0.2)$ m
and time $t = (4.0 \pm 0.3)$ s

$$\text{Velocity } \nu = \frac{s}{t} = \frac{13.8}{4.0} = 3.45\,\text{m s}^{-1} = 3.5\,\text{m s}^{-1}$$

$$\frac{\Delta\nu}{\nu} = \pm\left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right)$$

$$= \pm\left(\frac{0.8 + 4.14}{13.8\times 40.}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right)$$

$$\therefore \Delta\nu = \pm 0.0895 \times V = \pm 0.0895 \times 3.45$$

$$= \pm 0.3087 = \pm 0.31$$

Hence $\nu = (3.5 \pm 0.31)$ m s^{-1}

$$\text{Percentage error in velocity} = \frac{\Delta\nu}{\nu}\times 100$$

$$= \pm 0.0895 \times 100$$

$$= \pm 8.95\% = \pm 9\%$$

Measuring Instrument

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement first of all.

Instruments used in measurement are called measuring instruments.

Least Count: The least value of a quantity, which the instrument can measure accurately, is called the least count of the instrument.

Error: The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, has an error.

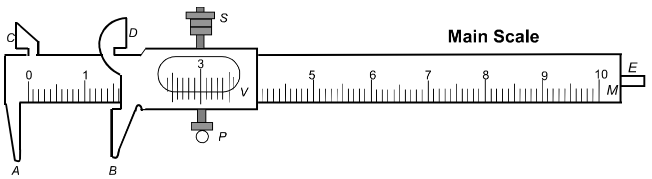
Accuracy and Precision: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

Vernier Callipers

It is a device used to measure accurately upto 0.1 mm. There are two scales in the vernier callipers, vernier scale and main scale. The main scale is fixed whereas the vernier scale is movable along the main scale.

Its main parts are as follows:

Main scale: It consists of a steel metallic strip *M*, graduated in cm and *mm* at one edge and in inches and tenth of an inch at the other edge on same side. It carries fixed jaws *A* and *C* projected at right angle to the scale as shown in figure.



Vernier Scale: A vernier *V* slides on the strip *M*. It can be fixed in any position by screw *S*. It is graduated on both sides. The side of the vernier scale which slides over the *mm* side has ten divisions over a length of 9 mm, i.e., over 9 main scale divisions and the side of the vernier scale which slides over the inches side has 10 divisions over a length of 0.9 inch, i.e., over 9 main scale divisions.

Movable Jaws: The vernier scale carries jaws *B* and *D* projecting at right angle to the main scale. These are called movable jaws. When vernier scale is pushed towards *A* and *C*, then as *B* touches *A*, straight side of *D* will touch straight side of *C*. In this position, in case of an instrument free from errors, zeros of vernier scale will coincide with zeros of main scales, on both the cm and inch scales.

(The object whose length or external diameter is to be measured is held between the jaws *A* and *B*, while the straight edges of *C* and *D* are used for measuring the internal diameter of a hollow object).

Metallic Strip: There is a thin metallic strip *E* attached to the back side of *M* and connected with vernier scale. When the jaws *A* and *B* touch each other, the edge of strip *E* touches the edge of *M*. When the jaws *A* and *B* are separated, *E* moves outwards. The strip E is used for measuring the depth of a vessel.

Determination of least count (Vernier Constant)

Note the value of the main scale division and count the number *n* of vernier scale divisions. Slide the movable jaw till the zero of vernier scale coincides with any of the mark of the main scale and find the number of divisions (*n* – 1) on the main scale coinciding with *n* divisions of vernier scale. Then

$$nV.S.D. = (n-1)\,M.S.D. \text{ or } 1\ V.S.D. = \left(\frac{n-1}{n}\right) M.S.D.$$

or $$V.C. = 1\ M.S.D. - 1\ V.S.D. = \left(1 - \frac{n-1}{n}\right) \text{M.S.D.}$$

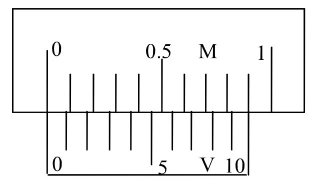
$$= \frac{1}{n} \text{ M.S.D.}$$

Determination of zero error and zero correction

For this purpose, movable jaw *B* is brought in contact with fixed jaw *A*.

One of the following situations will arise.

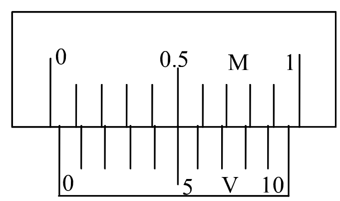
(i) Zero of Vernier scale coincides with zero of main scale (see figure)



In this case, zero error and zero correction, both are nil.

Actual length = observed (measured) length.

(ii) Zero of vernier scale lies on the right of zero of main scale (see figure)



Here 5th vernier scale division is coinciding with any main sale division.

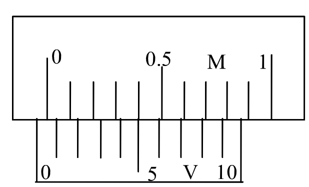
Hence, $N = 0$, $n = 5$, L.C. = 0.01 cm.

Zero error $N = n \times$ (L.C.) = 0 + 5 × 0.01 = + 0.05 cm

Zero correction = – 0.05 cm.

Actual length will be 0.05 cm less than the observed (measured) length.

(iii) zero of the vernier scale lies left of the main scale.



Here, 5th vernier scale division is coinciding with any main scale division.

In this case, zero of vernier scale lies on the right of – 0.1 cm reading on main scale.

Hence, $N = -0.1$ cm, $n = 5$, L.C. = 0.01 cm

Zero error = $N + n \times$ (L.C.) = – 0.1 + 5 × 0.01 = –0.05 cm.

Zero correction = +0.05 cm.

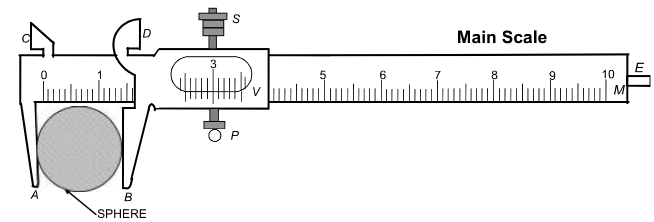
Actual length will be 0.05 cm more than the observed (measured) length.

Experiment

Aim: To measure the diameter of a small spherical/ cylindrical body, using a vernier callipers.

Apparatus: Vernier callipers, a spherical (pendulum bob) or a cylinder.

Diagram:



Theory: If with the body between the jaws, the zero of vernier scale lies ahead of Nth division of main scale, then main scale reading (M.S.R.) = N.

If n^{th} division of vernier scale coincides with any division of main scale, then vernier scale reading (V.S.R.)

$= n \times$ (L.C.) (L.C. is least count of vernier callipers)

$= n \times$ (V.C.) (V.C. is vernier constant of vernier callipers)

Total reading, $T.R. = M.S.R. + V.S.R. = N + n \times (V.C.)$

Precautions (to be taken)

1. Motion of vernier scale on main scale should be made smooth (by oiling if necessary).
2. Vernier constant and zero error should be carefully found and properly recored.
3. The body should be gripped between the jaws firmly but gently (without undue pressure on it from the jaws).
4. Observations should be taken at right angles at one place and taken at least at three different places.

Sources of Error

1. The vernier scale may be loose on main scale.
2. The jaws may not be at right angles to the main scale.
3. The graduations on scale may not be correct and clear.
4. Parallax may be there in taking observations.

SOLVED EXAMPLE

EXAMPLE 35

The least count of vernier callipers is 0.1 mm. The main scale reading before the zero of the vernier scale is 10 and the zeroth division of the vernier scale coincides with the main scale division. Given that each main scale division is 1 mm, what is the measured value?

Sol. Length measured with vernier callipers

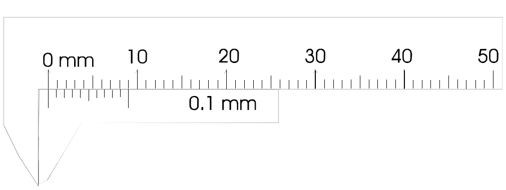
= reading before the zero of vernier scale + number of vernier divisions coinciding

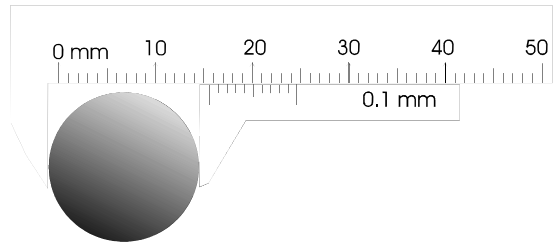
with any main scale division × least count

$= 10\text{ mm} + 0 \times 0.1\text{ mm} = 10\text{ mm} = \mathbf{1.00\ cm}$

EXAMPLE 36

Read the vernier





Sol. Thickness of the object = (main scale reading) + (vernier scale Reading) (least count)

where least count = (Main scale division - vernier Scale division)

= 1 mm - 0.9 mm (from figure)

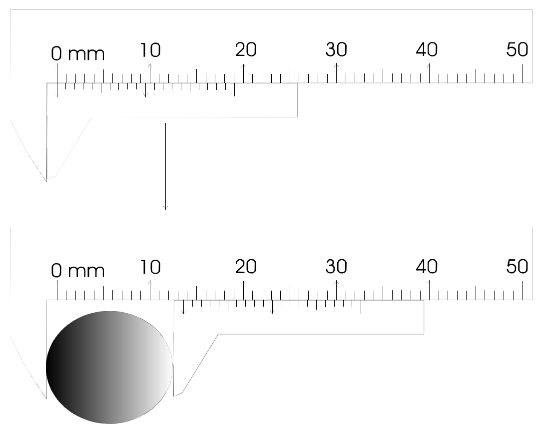
= 0.1 mm

So thickness of the object = 15 mm + (6) (0.1mm)

= 15.6 mm **Ans.**

EXAMPLE 37

Read the special type of vernier



Sol. Thickness of the object

= (main scale reading) + (vernier scale Reading) (least count)

where least count = (Main scale division – vernier Scale division)

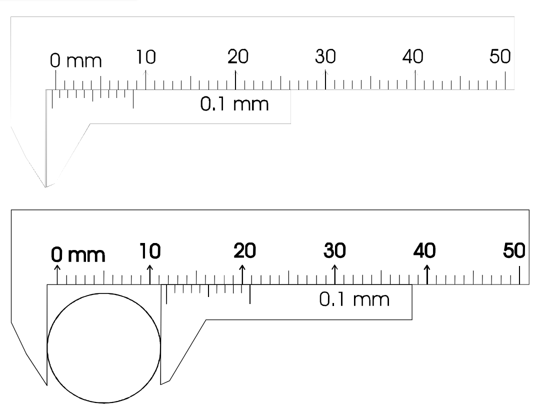
= 1 mm - 19/20 mm (from fig.)

= 0.05 mm

So thickness of the object = 13 mm + (12) (0.05mm)

= 13.60 mm **Ans.**

EXAMPLE 38



Sol. Zero error = main scale reading+(vernier scale reading)(least count)

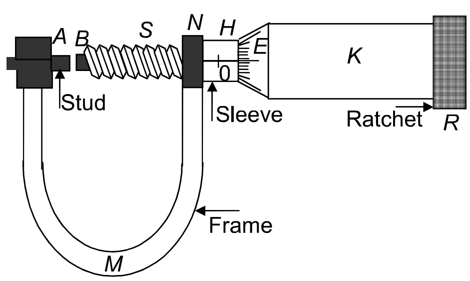
$= -1$ mm + 6 (0.1 mm) = -0.4 mm

observed reading = 11.8 mm

So actual thickness = 11.8 - (-0.4) = 12.2 mm

Screw Gauge

This instrument (shown in figure) works on the principle of micrometer screw. It consists of a U-shaped frame *M*. At one end of it is fixed a small metal piece *A* of gun metal. It is called stud and it has a plane face. The other end *N* of *M* carries a cylindrical hub *H*. The hub extends few millimetre beyond the end of the frame. On the tubular hub along its axis, a line is drawn known as reference line. On the reference line graduations are in millimetre and half millimeter depending upon the pitch of the screw. This scale is called linear scale or pitch scale. A nut is threaded through the hub and the frame *N*. Through the nut moves a screw *S* made of gun metal. The front face *B* of the screw, facing the plane face *A*, is also plane. A hollow cylindrical cap *K*, is capable of rotating over the hub when screw is rotated. It is attached to the right hand end of the screw. As the cap is rotated the screw either moves in or out. The bevelled surface *E* of the cap *K* is divided into 50 or 100 equal parts. It is called the circular scale or head scale. Right hand end *R* of *K* is milled for proper grip.



In most of the instrument the milled head *R* is not fixed to the screw head but turns it by a spring and ratchet arrangement such that when the body is just held between faces *A* and *B,* the spring yields and milled head *R* turns without moving in the screw.

In an accurately adjusted instrument when the faces *A* and *B* are just touching each other, the zero marks of circular scale and pitch scale exactly coincide.

Determination of least count of screw gauge

Note the value of linear (pitch) scale division. Rotate screw to bring zero mark on circular (head) scale on reference line. Note linear scale reading i.e. number of divisions of linear scale uncovered by the cap.

Now give the screw a few known number of rotations. (one rotation completed when zero of circular scale again arrives on the reference line). Again note the linear scale reading. Find difference of two readings on linear scale to find distance moved by the screw. Then, pitch of the screw

$$= \frac{\text{Distance moved by in n rotation}}{\text{No. of full rotation (n)}}$$

Now count the total number of divisions on circular (head) scale.

Then, least count

$$= \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}}$$

The least count is generally 0.001 cm.

Determination of zero error and zero correction

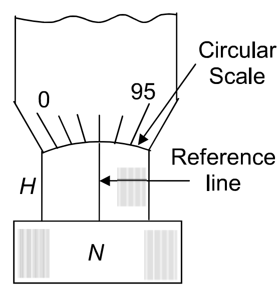
For this purpose, the screw is rotated forward till plane face *B* of the screw just touches the fixed plane face *A* of the stud and edge of cap comes on zero mark of linear scale. Screw gauge is held keeping the linear scale vertical with its zero downwards.

One of the following three situations will arise.

(i) **Zero mark of circular scale comes on the reference line** (see figure)

In this case, zero error and zero correction, both are nil

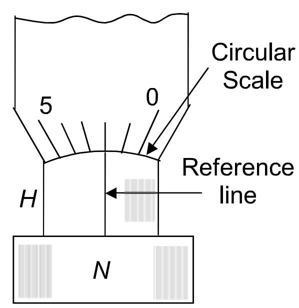
Actual thickness = Observed (measured) thickness.



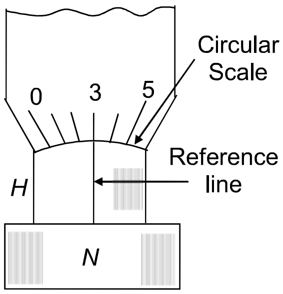
(ii) **Zero mark of circular scale remains on right of reference line and does not cross** it (see figure).

Here 2nd division on circular scale comes on reference line. Zero reading is already 0.02 mm. It makes zero error + 0.02 mm and zero correction – 0.02 mm.

Actual thickness will be 0.02 mm less than the observed (measured) thickness.



(iii) **Zero mark of circular scale goes to left on reference line after crossing it (see figure).** Here zero of circular scale has advanced from reference line by 3 divisions on circular scale. A backward rotation by 0.03 mm will make reading zero. It makes zero error – 0.03 mm and zero correction + 0.03 mm.



Actual thickness will be 0.03 mm more than the observed (measured) thickness.

Experiment

Aim: To measure diameter of a given wire using a screw gauge and find its volume.

Apparatus: Screw gauge, wire, half metre rod (scale).

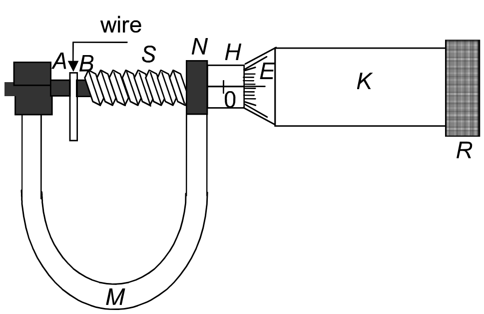
Theory:

(1) Determine of least count of screw gauge

(2) If with the wire between plane faces *A* and *B*, the edge of the cap lies ahead of N^{th} division of linear scale.
Then, linear scale reading (L.S.R.) = N
If nth division of circular scale lies over reference line.
Then, circular scale reading (C.S.R.) = $n \times$ (L.C.)
(L.C. is least count of screw gauge)
Total reading (T.R.) = L.S.R. + C.S.R. = $N + n \times$ (L.C.)

(3) If D be the mean diameter and l be the mean length of the wire. Then volume of the wire, $V = \pi\left(\frac{D}{2}\right)^2 l$

Diagram



Calculation

Mean diameter of the wire,

$$D = \frac{D_1(a) + D_1(b) + ... + D_5(a) + D_5(b)}{10} = ...\text{mm} = \ldots \text{cm}$$

Mean length of the wire,

$$l = \frac{l_1 + l_2 + l_3}{3} = ...\text{cm}$$

Volume of the wire

$$V = \pi\left(\frac{D}{2}\right)^2 l = \ldots \text{cm}^3$$

Result The volume of the given wire is = … cm^3

Precaution (to be taken)

1. While taking an observation, the screw must always be turned only in one direction so as to avoid the backlash error.
2. At each place, take readings in pairs i.e. in two directions at right angles to each other.
3. The wire must be straight and free from kinks.
4. Always rotate the screw by the ratchet and stop as soon as it gives one tick sound only.
5. While taking a reading, rotate the screw in only one direction so as to avoid the backlash error.

Sources of Error

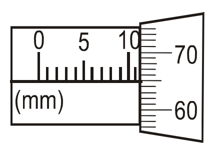
1. The screw may have friction.
2. The screw gauge may have back-lash error.
3. Circular scale divisions may not be of equal size.
4. The wire may not be uniform.

SOLVED EXAMPLE

EXAMPLE 39

Read the normal screwgauge
*Main scale has only mm marks.
*Circular scale has 100 divisions
*In complete rotation, the screw advances by 1 mm.

Sol.



Soln: Object thickness = 11 mm + 65 $\left(\frac{1 \text{ mm}}{100}\right)$
= 11.65 mm

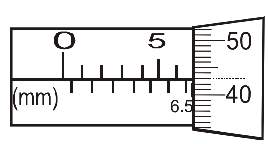
EXAMPLE 40

Read the screwgauge

*Main scale has $\frac{1}{2}$ mm marks.

*Circular scale has 50 division.
*In complete rotation, the screw advances by $\frac{1}{2}$ mm.

Sol.



Soln: Object thickness = 6.5 - - - -
Object thickness = 6.5 mm + 43 $\left(\frac{1/2 \text{ mm}}{50}\right)$
= 6.93 mm

EXAMPLE 41

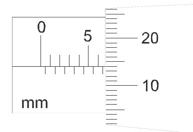
Read the screwgauge shown bellow:

*Main scale has $\frac{1}{2}$ mm marks.

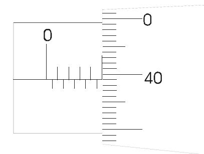
*Circular scale has 50 division.

*In complete rotation, the screw advances by $\frac{1}{2}$ mm.

Sol.



Object thickness = 6.5 mm + 14 $\left(\frac{1/2\text{ mm}}{50}\right)$



Object thickness = 4.5 mm + 39 $\left(\frac{1/2\text{ mm}}{50}\right)$
= 4.89 mm

DPP-3

Q.1 The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
(1) 5, 1, 2
(2) 5, 1, 5
(3) 5, 5, 2
(4) 4, 4, 2

Q.2 The edge of a cube is $a = 1.2 \times 10^{-2}$ m. Then its volume will be recorded as :
(1) $1.72 \times 10^{-6}\,m^3$
(2) $1.728 \times 10^{-6}\,m^3$
(3) $1.7 \times 10^{-6}\,m^3$
(4) $1.73 \times 10^{-6}\,m^3$

CLASS ASSIGNMENT

Q.1 The time dependence of a physical quantity p is given by $p = p_0 \exp(-at^2)$, where a is a constant and t is the time. The constant a
(1) is dimensionless
(2) has dimensions $[T^{-2}]$
(3) has dimensions $[T^2]$
(4) has dimensions of p

Q.2 A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of :
(1) volume (2) energy
(3) momentum (4) area

Q.3 Find dimensional formula of λ if $\lambda = \frac{C}{V}$ (where, C = capacity; V = Voltage)
(1) $[M^{-3}L^{-3}T^7A^3]$ (2) $[M^{-2}L^{-2}T^7A^3]$
(3) $[M^{-1}L^{-2}T^7A^{-3}]$ (4) $[M^{-2}L^{-4}T^7A^3]$

Q.4 Dimensions of $1/(\mu_0 \in_0)$ where symbols have their usual meaning are -
(1) L^2T^{-2} (2) L^2T^2
(3) $L^{-1}T$ (4) LT^{-1}

Q.5 Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?
(1) H/m^2 (2) Weber (Wb)
(3) Wb/m^2 (4) Henry (H)

Q.6 If p is radiation pressure, c represents speed of light and Q represents radiation energy striking unit area per second, then non zero integers x, y, and z such that $P^x Q^y C^z$ is dimensionless are:
(1) x = 1, y = 1, z = –1 (2) x = 1, y = –1, z = 1
(3) x = –1, y = 1, z = 1 (4) x = 1, y = 1, z = 1

Q.7 The density of material in CGS system of unit is 4 g/cm^3. In a system of units in which unit of length is 10 cm and unit of mass is 100g, the value of density of material will be:
(1) 0.4 (2) 40
(3) 400 (4) 0.04

Q.8 Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be
(1) $[M] = [TC^{-2} h]$ (2) $[M] = [T^{-1} C^{-2} h^{-1}]$
(3) $[M] = [T^{-1} C^{-2} h]$ (4) $[M] = [T^{-1} C^2 h]$

Q.9 In a given system of unit standard measurment of mass is 100 gm, standard measurement of length is 200 cm and standard measurement of time is 5 sec. 10 J energy in the given system of unit has value N,then value of N is.
(1) 625 (2) 525
(3) 125 (4) 25

Q.10 During a short interval of time, speed v in m/s of an automobile is given by $v = at^2 + bt^3$, where the time t is in seconds. The units of a and b are respectively:
(1) $m \cdot s^2$; $m \cdot s^4$ (2) s^3/m; s^4/m
(3) m/s^2; m/s^3 (4) m/s^3; m/s^4

Q.11 Given that $ln(\alpha/p\beta) = \alpha z/K_B\theta$ where p is pressure, z is distance, K_B is Boltzmann constant and θ is temperature, the dimensions of β are (useful formula Energy = K_B × temperature)
(1) $L^0M^0T^0$ (2) $L^1M^{-1}kT^2$
(3) $L^2M^0T^0$ (4) $L^{-1}M^1T^{-2}$

Q.12 The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :
(1) 3.5 % (2) 4.5 %
(3) 6 % (4) 2.5 %

Q.13 The radius of a ball is (5.4 ± 0.2) cm. The percentage error in the volume of the ball is
(1) 11% (2) 4%
(3) 7% (4) 9%

Q.14 In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of $\frac{L}{T^2}$ is
(1) 5% (2) 7%
(3) 8% (4) 1%

Q.15 If measurement of x is equal to 100 ± 6, then find the value of $\sqrt{x}$.
(1) 10.0 ± 0.3 (2) 10.0 ± 0.5
(3) 10 ± 6 (4) 10 ± 2

Q.16 The period of oscillation of a simple pendulum in the experiment is recorded as 2.63s, 2.56s, 2.42s, 2.71s and 2.80s respectively. The average absolute error is
(1) 0.1s (2) 0.11s
(3) 0.01s (4) 1.0s

Q.17 A physical quantity $P = \frac{\sqrt{abc^2}}{d^3}$ is determined by measuring a, b, c and d separately with percentage error of 2%, 3%, 2% and 1% respectively. Minimum amount of error is contributed by the measurement of
(1) b (2) a
(3) d (4) c

Q.18 Choose the incorrect statement
(1) 0.0037218 has five significant digits
(2) 4.3500 has three significant digits
(3) 1560 has three significant digits
(4) 7.650 has four significant digits

Q.19 The length and breadth of a metal sheet are 2.214 m and 2.002 m respectively. The area of this sheet up to four correct significant figures is
(1) 4.43 m^2 (2) 4.432 m^2
(3) 4.4324 m^2 (4) 4.432428 m^2

Q.20 Which of the following has the least number of significant figures ?
(1) 1.64×10^{20} kg (2) 0.006 m^2
(3) 7.2180 J (4) 5.045 J

Q.21 The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed ?
(1) 11 % (2) 8 %
(3) 5 % (4) 1 %

Q.22 The pitch of a screw guage is 0.1 cm. To measure up to accuracy of 0.0005 cm , number of divisions are required on circular scale.
(1) 200 (2) 400
(3) 50 (4) 100

Q.23 In a vernier calliper, N divisions of vernier scale coincide with (N – 1) divisions of main scale (in which 1 division represents 1mm). The least count of the instrument in cm. should be
(1) N (2) N – 1 (3) $\frac{1}{10N}$ (4) $\frac{1}{N-1}$

Q.24 In a vernier callipers having 10 vsd, the least count is 0.1 mm. When the jaws are closed, zero of vernier lies to the left of zero of main and 7^{th} vsd coincides with a main scale division. When a cylinder is placed between the jaws the main scale reading was 7.7 cm and vernier scale read 8 divisions. What is the diameter of the cylinder ?
(1) 78.1 mm (2) 77.5 mm
(3) 77.8 mm (4) 78.5 mm

Q.25 The length of a cylinder is measured with a metre rod having least count 0.1 cm. Its diameter is measured with vernier callipers having least count 0.01 cm. Given the length is 5.0 cm. and radius is 2.00 cm. The percentage error in the calculated value of volume will be –
(1) 2% (2) 1%
(3) 3% (4) 4%

Q.26 A vernier callipers has 20 divisions on the vernier scale which coincide with 19 divisions on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of
(1) 0.5 mm (2) 1 mm
(3) 2 mm (4) 1/4 mm

Q.27 PARSEC is a unit of
(1) Time (2) Angle
(3) Distance (4) Velocity

Q.28 The unit of magnetic moment is-
(1) amp m^2 (2) amp m^{-2}
(3) amp m (4) amp m^{-1}

Q.29 Which of the following statement is wrong ?
(1) Unit of K.E. is Newton-metre
(2) Unit of viscosity is poise
(3) Work and energy have same dimensions
(4) Unit of surface tension is Newton metre

Q.30 The SI unit of Stefan's constant is :
(1) $W s^{-1} m^{-2} K^{-4}$ (2) $J s m^{-1} K^{-1}$
(3) $J s^{-1} m^{-2} K^{-1}$ (4) $W m^{-2} K^{-4}$

Q.31 A pair of physical quantities having the same dimensional formula is :
(1) angular momentum and torque
(2) torque and energy
(3) force and power
(4) power and angular momentum

Q.32 $\int \frac{x\,dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$. The value of n is :
You may use dimensional analysis to solve the problem.
(1) 0 (2) –1
(3) 1 (4) none of these

Q.33 An unknown quantity "α" is expressed as $\alpha = \frac{2ma}{\beta} \log\left(1 + \frac{2\beta\ell}{ma}\right)$ where m = mass, a = acceleration, ℓ = length. The unit of α should be
(1) meter (2) m/s (3) m/s^2 (4) s^{-1}

Q.34 The random error in the arithmetic mean of 100 observations is x; then random error in the arithmetic mean of 400 observations would be

(1) $4x$ (2) $\frac{1}{4}x$ (3) $2x$ (4) $\frac{1}{2}x$

Q.35 If E, M, J and G denote energy, mass, angular momentum and gravitational constant respectively, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of

(1) length (2) angle
(3) mass (4) time

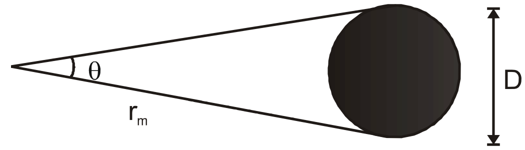
Q.36 The position of a particle at time 't' is given by the relation $x(t) = \frac{V_0}{\alpha}[1 - e^{-\alpha t}]$ where V_0 is a constant and $\alpha > 0$. The dimensions of V_0 and α are respectively.

(1) $M^0L^1T^0$ and T^{-1} (2) $M^0L^1T^0$ and T^{-2}
(3) $M^0L^1T^{-1}$ and T^{-1} (4) $M^0L^1T^{-1}$ and T^{-2}

Q.37 If area (A) velocity (v) and density (ρ) are base units, then the dimensional formula of force can be represented as

(1) $Av\rho$ (2) $Av^2\rho$
(3) $Av\rho^2$ (4) $A^2v\rho$

Q.38 The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the act that the moon is about 384000 km away to find the approximate diameter of the moon.



(1) 192000 km (2) 3350 km
(3) 1600 km (4) 1920 km

Q.39 The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively (Obviously). Maximum permissible error in area measurement is -

(1) $\pm$ 0.2 cm^2
(2) $\pm$ 0.1 cm^2
(3) $\pm$ 0.3 cm^2
(4) Zero

Q.40 In the previous question, minimum possible error in area measurement can be -

(1) $\pm$ 0.02 cm^2 (2) $\pm$ 0.01 cm^2
(3) $\pm$ 0.03 cm^2 (4) Zero

HOME ASSIGNMENT

Q.1 The unit of permittivity of free space, ε_0, is

(1) coulomb/newton-meter
(2) newton-meter^2/ coulomb^2
(3) coulomb^2/newton-meter^2
(4) coulomb^2/(newton-meter)^2

Q.2 The dimensions of solar constant (energy falling on earth per second per unit area) are

(1) $[M^0L^0T^0]$ (2) $[MLT^{-2}]$
(3) $[ML^2T^{-2}]$ (4) $[MT^{-3}]$

Q.3 The dimension of the physical quantity α in the equation, $E = V\alpha$, where E is the energy and V is volume has same dimension as of

(1) Impulse
(2) Power
(3) Linear Momentum
(4) Pressure

Q.4 The pairs having same dimensional formula -

(1) Angular momentum, torque
(2) Torque, work
(3) Plank's constant, boltzman's constant
(4) Gas constant, pressure

Q.5 The dimensionally correct expression for the resistance R among the following is [P = electric power, I = electric current, t = time, V = voltage and E = electric energy]

(1) $R = \sqrt{PI}$ (2) $R = \frac{E}{I^2t}$
(3) $R = V^2P$ (4) $R = VI$

Q.6 In a particular system, the unit of length, mass and time are chosen to be 10 cm, 10g and 0.1s respectively. The unit of force in this system will be equivalent to -

(1) 1/10 N (2) 1 N (3) 10 N (4) 100 N

Q.7 If the units of length and force are increased four times, by what factor is the unit of energy is increased?

(1) 16 times (2) 4 times
(3) 2 times (4) None of these

Q.8 If C, the velocity of light, g the acceleration due to gravity and P the atmospheric pressure be the fundamental quantities in MKS system, then the dimensions of length will be same as that of

(1) $\frac{C}{g}$ (2) $\frac{C}{P}$ (3) PCg (4) $\frac{C^2}{g}$

Q.9 The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$. The dimensions of a and d are -
(1) L, T^{-3} (2) L, LT^{-3}
(3) L^0, T^3 (4) none of these

Q.10 A substance of mass 49.53 g occupies 1.5 cm^3 of volume. The density of the substance (in g cm^{-3}) with correct number of significant figure is
(1) 3.3 (2) 3.300
(3) 3.302 (4) 3.30

Q.11 The resistance is $R = \frac{V}{I}$ where $V = 100 \pm 5$ Volts and $I = 10 \pm 0.2$ amperes. What is the total error in R ?
(1) 5 % (2) 7 %
(3) 5.2 % (4) $\left(\frac{5}{2}\right)$ %

Q.12 The length, breadth and thickness of a strip are (10.0 ± 0.1) cm, (1.00 ± 0.01)cm and (0.100 ± 0.001)cm respectively. The most probable error in its volume will be
(1) ± 0.03 cm^3 (2) ± 0.111 cm^3
(3) ± 0.012 cm^3 (4) None of these

Q.13 Percentage error in measuring the radius and mass of a solid sphere are 2% & 1% respectively. Then error in measurement of moment of inertia with respect to its diameter is :-
(1) 3 % (2) 6 % (3) 5 % (4) 4 %

Q.14 One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is
(1) 0.01 cm (2) 0.02 cm
(3) 0.05 cm (4) 0.005 cm

Q.15 A student measured the diameter of a wire using a screw gauge with least count 0.001 cm and listed the measurements. The correct measurement is –
(1) 5.3 cm (2) 5.32 cm
(3) 5.320 cm (4) 5.3200 cm

Q.16 A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
(1) A meter scale.
(2) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(3) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(4) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

Q.17 A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as
(1) (5.5375 ± 0.0739) mm
(2) (5.54 ± 0.07) mm
(3) (5.538 ± 0.074) mm
(4) (5.5375 ± 0.0740) mm

Q.18 Which of the following sets cannot enter into the list of fundamental quantities in any system of units?
(1) length, mass and velocity
(2) length, time and velocity
(3) mass, time and velocity
(4) length, time and mass

Q.19 A unit less quantity
(1) never has a nonzero dimension
(2) always has a nonzero dimension
(3) may have a nonzero dimension
(4) does not exist

Q.20 Which of the following is not the name of a physical quantity ?
(1) kilogram (2) impulse
(3) energy (4) density

Q.21 Light year is the unit of
(1) speed (2) mass
(3) distance (4) time

Q.22 Which of the following system of units is NOT based on the unit of mass, length and time alone
(1) FPS (2) SI
(3) CGS (4) MKS

Q.23 In the S.I. system, the unit of energy is-
(1) erg (2) calorie
(3) joule (4) electron volt

Q.24 Unit of pressure in S.I. system is-
(1) atmosphere (2) dynes per square cm
(3) pascal (4) bar

Q.25 The SI unit of the universal gravitational constant G is
(1) $Nm\,kg^{-2}$ (2) Nm^2kg^{-2}
(3) $Nm^2\,kg^{-1}$ (4) $Nmkg^{-1}$

Q.26 Surface tension has unit of-
(1) Joule m^2 (2) Joule m^{-2}
(3) Joule m^{-1} (4) Joule m^3

Q.27 The unit of intensity of magnetisation is-
(1) Amp m^2 (2) Amp m^{-2}
(3) Amp m (4) Amp m^{-1}

Q.28 The M.K.S. units of coefficient of viscosity is-
(1) kg $m^{-1}s^{-1}$ (2) kg m s^{-2}
(3) kg m^2 s^{-1} (4) kg^{-1} m^{-1} s^2

Q.29 The specific resistance has the unit of-
(1) ohm/m (2) ohm/m^2
(3) ohm-m^2 (4) ohm-m

Q.30 The mutual inductance has unit of-
(1) Gauss (2) Weber
(3) Farad (4) Henry

Q.31 In SI unit, the angular acceleration has unit of-
(1) $Nmkg^{-1}$ (2) ms^{-2}
(3) rad s^{-2} (4) Nkg^{-1}

Q.32 What is the exponent of length in force × displacement/ time
(1) –2 (2) 0
(3) 2 (4) none of these

Q.33 The angular frequency is measured in rad s^{-1}. Its exponent in length is:
(1) – 2 (2) –1
(3) 0 (4) 2

Q.34 [M L T^{-1}] are the dimensions of-
(1) power (2) momentum
(3) force (4) couple

Q.35 The dimensional formula for angular momentum is-
(1) ML^2T^{-2} (2) ML^2T^{-1}
(3) MLT^{-1} (4) $M^0L^2T^{-2}$

Q.36 What are the dimensions of Boltzmann's constant?
(1) $MLT^{-2}K^{-1}$ (2) $ML^2T^{-2}K^{-1}$
(3) M^0LT^{-2} (4) $M^0L^2T^{-2}K^{-1}$

Q.37 Dimensions of magnetic flux density is -
(1) $M^1 L^0 T^{-1}A^{-1}$ (2) $M^1 L^0 T^{-2}A^{-1}$
(3) $M^1 L^1 T^{-2}A^{-1}$ (4) $M^1 L^0 T^{-1}A^{-2}$

Q.38 The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as (where k is a dimensionless constant)
(1) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$ (2) $v^2 = k\, g\, \lambda$
(3) $v^2 = k\, g\, \lambda\, \rho$ (4) $v^2 = k\, \lambda^3 g^{-1} \rho^{-1}$

Q.39 Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross–sectional area of the stream (A). The expression of the force should be
(1) ρAv (2) ρAv^2
(3) ρ^2Av (4) ρA^2v

Q.40 Given that v is the speed, r is radius and g is acceleration due to gravity. Which of the following is dimensionless
(1) $\frac{v^2g}{r}$ (2) v^2rg
(3) vr^2g (4) $\frac{v^2}{rg}$

Q.41 The value of G = 6.67 × 10^{-11} N m^2 $(kg)^{-2}$. Its numerical value in CGS system will be :
(1) 6.67 × 10^{-8} (2) 6.67 × 10^{-6}
(3) 6.67 (4) 6.67 × 10^{-5}

Q.42 If unit of length and time is doubled, the numerical value of 'g' (acceleration due to gravity) will be :
(1) doubled (2) halved
(3) four times (4) remain same

Q.43 A physical quantity is measured and the result is expressed as nu where u is the unit used and n is the numerical value. If the result is expressed in various units then
(1) $n \propto u$ (2) $n \propto u^2$ (3) $n \propto \sqrt{u}$ (4) $n \propto 1/u$

Q.44 One watt-hour is equivalent to
(1) 6.3 × 10^3 Joule (2) 6.3 × 10^{-7} Joule
(3) 3.6 × 10^3 Joule (4) 3.6 × 10^{-3} Joule

Q.45 The pressure of 10^6 dyne/cm^2 is equivalent to
(1) 10^5 N/m^2 (2) 10^6 N/m^2
(3) 10^7 N/m^2 (4) 10^8 N/m^2

Q.46 The SI unit of length is meter. Suppose we adopt a new unit of length which equals to x meter. The area 1m^2 expressed in terms of the new unit has a magnitude-
(1) x (2) x^2
(3) $\frac{1}{x}$ (4) $\frac{1}{x^2}$

Q.47 ρ = 2 g/cm^3 convert it into MKS system -
(1) $2 \times 10^{-3}\ \frac{kg}{m^3}$ (2) $2 \times 10^{3}\ \frac{kg}{m^3}$
(3) $4 \times 10^{3}\ \frac{kg}{m^3}$ (4) $2 \times 10^{6}\ \frac{kg}{m^3}$

Q.48 The density of mercury is 13600 kg m^{-3}. Its value of CGS system will be :
(1) 13.6 g cm^{-3} (2) 1360 g cm^{-3}
(3) 136 g cm^{-3} (4) 1.36 g cm^{-3}

Q.49 The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of the kinetic energy obtained by measuring mass and speed
(1) 11% (2) 8%
(3) 5% (4) 1%

Q.50 If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of
(1) velocity
(2) displacement
(3) acceleration
(4) momentum

Q.51 When a wave traverses a medium, the displacement of a particle located at x at time t is given by y = a sin (bt – cx) where a, b and c are constants of the wave. The dimensions of b are the same as those of
(1) wave velocity
(2) amplitude
(3) wavelength
(4) wave frequency

Q.52 In the above question dimensions of $\frac{b}{c}$ are the same as those of
(1) wave velocity
(2) wavelength
(3) wave amplitude
(4) wave frequency

Q.53 In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2kl}{ma}}\right]$ here m represents mass, a represents accelerations, *l* represents length. The unit of b should be
(1) m/s (2) m/s^2
(3) meter (4) /sec

Q.54 If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:
(1) FT^2
(2) $F^{-1} A^2 T^{-1}$
(3) FA^2T
(4) AT^2

Statement Type Questions(Q.No. 55 to 57)
(1) Both statement I and statement II are correct.
(2) Statement I is correct and statement II is incorrect.
(3) Statament I is incorrect and statement II is correct
(4) Both statements I and statements II are incorrects.

Q.55 **Statements I :** In a measurement, two reading obtained are 20.004 and 20.0004. The second measurement is more precise.
Statements II : Measurement having more decimal places is more precise.
In the light of the above statements, choose the most appropriate answer from the options given below:

Q.56 **Statement I :** Absolute error is unitless and dimensionless.
Statement II : All types of errors are unitless and dimesnsionless.

Q.57 **Statements I :** Least count of a screw gauge is directly proportional to the number of divisions on circular scale.
Statement II : A screw gauge having a smaller value of pitch has greater accurancy.
In the light of the above statements, choose the most appropriate answer from the options given below:

Assertion &Reason Type Questions (Q.No. 58 to 60)
(1) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(2) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(3) If assertion is true but reason is false.
(4) If the assertion and reason both are false.

Q.58 **Assertion (A) :** 'Ligth year' and 'Wavelength' both measure distance.
Reason (R) : Both have dimensions of time.

Q.59 **Assertion (A) :** Force cannot be added to pressure.
Reason (R) : Because their dimensions are different.

Q.60 **Assertion (A) :** Linear mass density has the dimensions of $[M^1L^{-1}T^0]$.
Reason (R) : Because density is always mass per unit volume.

NEET PREVIOUS YEAR'S

Q.1 Planck's constant (h), speed of light in vacuum (c) and Newton's gravitational constant(G) are three fundamental constant. Which of the following combinations of these has the dimension of length? **[NEET Phase II-2016]**

(1) $\frac{\sqrt{hG}}{c^{3/2}}$ (2) $\frac{\sqrt{hG}}{c^{5/2}}$

(3) $\sqrt{\frac{hc}{G}}$ (4) $\sqrt{\frac{Ge}{h^{3/2}}}$

Q.2 A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is the universal constant of gravitation and e is charge] **[NEET-2017]**

(1) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (2) $\frac{1}{c^2}\left[\frac{e^2}{G\,4\pi\varepsilon_0}\right]^{1/2}$

(3) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$ (4) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$

Q.3 The unit of thermal conductivity is : **[NEET-2019]**

(1) $J\,m\,K^{-1}$ (2) $J\,m^{-1}\,K^{-1}$
(3) $W\,m\,K^{-1}$ (4) $W\,m^{-1}\,K^{-1}$

Q.4 Dimensions of stress are : **[NEET-2020]**

(1) $[ML^2T^{-2}]$ (2) $[ML^0T^{-2}]$
(3) $[ML^{-1}T^{-2}]$ (4) $[MLT^{-2}]$

Q.5 If E and G respectively denote energy and gravitational constant, then $\frac{E}{G}$ has the dimensions of **[NEET-2021]**

(1) $[M][L^{-1}][T^{-1}]$ (2) $[M][L^0][T^0]$
(3) $[M^2][L^{-2}][T^{-1}]$ (4) $[M^2][L^{-1}][T^0]$

Q.6 If force [F] acceleration [A] and time [T] are chosen as the fundamental physical quantities. Find the dimensions of energy. **[NEET-2021]**

(1) $[F]\,[A]\,[T^2]$ (2) $[F]\,[A]\,[T^{-1}]$
(3) $[F]\,[A^{-1}]\,[T]$ (4) $[F]\,[A]\,[T]$

Q.7 The area of a rectangular field (in m^2) of length 55.3 m and breadth 25 m after rounding off the value for correct significant digits is: **[NEET-2022]**

(1) 1382 (2) 1382.5
(3) 14×10^2 (4) 138×10^1

Q.8 The dimensions $[MLT^{-2}A^{-2}]$ belong to the : **[NEET-2022]**

(1) Self inductance
(2) Magnetic permeability
(3) Electric permittivity
(4) Magnetic flux

Q.9 Plane angle and solid angle have : **[NEET-2022]**

(1) Dimensions but no units
(2) No units and no dimensions
(3) Both units and dimensions
(4) Units but no dimensions

Q.10 The quantities which have the same dimensions as those of solid angle are: **[NEET-2024]**

(1) angular speed and stress
(2) strain and angle
(3) stress and angle
(4) strain and arc

Q.11 A force defined by $F = \alpha t^2 + \beta t$ acts on a particle at a given time t. The factor which is dimensionless, if α and β are constants, is: **[NEET-2024]**

(1) $\frac{\alpha\beta}{t}$ (2) $\frac{\beta t}{\alpha}$

(3) $\frac{\alpha t}{\beta}$ (4) $\alpha\beta t$

ERRORS IN MEASUREMENT

Q.12 A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and 25 divisions above the reference level. If screw gauge has a zero error of – 0.004 cm, the correct diameter of the ball is **[NEET 2018]**

(1) 0.521 cm
(2) 0.525 cm
(3) 0.053
(4) 0.529 cm

Q.13 In an experiment, the percentage of error occurred in the measurment of physical quantities A, B, C and D are 1%, 2%, 3% and 4% respectively. Then the maximum percentage of error in the measurement X, where

$X = \frac{A^2 B^{1/2}}{C^{1/3} D^3}$, will be : **[NEET-2019]**

(1) $\left(\frac{3}{2}\right)\%$ (2) 16% (3) –10% (4) 10%

Q.14 A screw gauge has least count of 0.01 mm and there are 50 divisions in its circular scale. The pitch of the screw gauge is : **[NEET-2020]**

(1) 0.25 mm (2) 0.5 mm
(3) 1.0 mm (4) 0.01 mm

Q.15 Taking into account of the significant figures, what is the value of 9.99 m – 0.0099 m? **[NEET-2020]**

(1) 9.98 m (2) 9.980 m
(3) 9.9 m (4) 9.9801 m

Q.16 A screw gauge gives the following readings when used to measure the diameter of a wire
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale correspond to 100 divisions on the circular scale. The diameter of the wire from the above data is :
[NEET-2021]

(1) 0.026 cm (2) 0.26 cm
(3) 0.052 cm (4) 0.52 cm

Q.17 A metal wire has mass (0.4 ± 0.002) g, radius (0.3 ± 0.001) mm and length (5 ± 0.02) cm. The maximum possible percentage error in the measurement of density will nearly be : **[NEET-2023]**

(1) 1.3% (2) 1.6%
(3) 1.4% (4) 1.2%

Q.18 The errors in the measurement which arise due to unpredictable fluctuations in temperature and voltage supply are : **[NEET-2023]**

(1) Personal errors
(2) Least count errors
(3) Random errors
(4) Instrumental errors

Q.19 In a vernier callipers, (N+1) divisions of vernier scale coincide with N divisions of main scale. If 1 MSD represents 0.1 mm, the vernier constant (in cm) is :
[NEET-2024]

(1) 10 (N + 1) (2) $\frac{1}{10N}$

(3) $\frac{1}{100(N+1)}$ (4) 100 N

ANSWER KEY

DPP-1

Q.1 (4) **Q.2** (4) **Q.3** (3) **Q.4** (1) **Q.5** (4)

DPP-2

Q.1 (4) **Q.2** (1) **Q.3** (4) **Q.4** (2) **Q.5** (3) **Q.6** (1) **Q.7** (1) **Q.8** (1) **Q.9** (3) **Q.10** (4)

DPP-3

Q.1 (1) **Q.2** (3)

CLASS ASSIGNMENT

Q.1 (2) **Q.2** (2) **Q.3** (4) **Q.4** (1) **Q.5** (4) **Q.6** (2) **Q.7** (2) **Q.8** (3) **Q.9** (1) **Q.10** (4)
Q.11 (3) **Q.12** (2) **Q.13** (1) **Q.14** (2) **Q.15** (1) **Q.16** (2) **Q.17** (2) **Q.18** (2) **Q.19** (2) **Q.20** (2)
Q.21 (2) **Q.22** (1) **Q.23** (3) **Q.24** (1) **Q.25** (3) **Q.26** (3) **Q.27** (3) **Q.28** (1) **Q.29** (4) **Q.30** (4)
Q.31 (2) **Q.32** (3) **Q.33** (1) **Q.34** (2) **Q.35** (2) **Q.36** (3) **Q.37** (2) **Q.38** (2) **Q.39** (1) **Q.40** (4)

HOME ASSIGNMENT

Q.1 (3) **Q.2** (4) **Q.3** (4) **Q.4** (2) **Q.5** (2) **Q.6** (1) **Q.7** (1) **Q.8** (4) **Q.9** (2) **Q.10** (1)
Q.11 (2) **Q.12** (1) **Q.13** (3) **Q.14** (2) **Q.15** (3) **Q.16** (2) **Q.17** (2) **Q.18** (2) **Q.19** (1) **Q.20** (1)
Q.21 (3) **Q.22** (2) **Q.23** (3) **Q.24** (3) **Q.25** (2) **Q.26** (2) **Q.27** (4) **Q.28** (1) **Q.29** (4) **Q.30** (4)
Q.31 (3) **Q.32** (3) **Q.33** (3) **Q.34** (2) **Q.35** (2) **Q.36** (2) **Q.37** (2) **Q.38** (2) **Q.39** (2) **Q.40** (4)
Q.41 (1) **Q.42** (1) **Q.43** (4) **Q.44** (3) **Q.45** (1) **Q.46** (4) **Q.47** (2) **Q.48** (1) **Q.49** (2) **Q.50** (4)
Q.51 (4) **Q.52** (1) **Q.53** (3) **Q.54** (4) **Q.55** (1) **Q.56** (4) **Q.57** (3) **Q.58** (3) **Q.59** (1) **Q.60** (2)

NEET PREVIOUS YEAR'S

Q.1 (1) **Q.2** (4) **Q.3** (4) **Q.4** (3) **Q.5** (4) **Q.6** (1) **Q.7** (3) **Q.8** (2) **Q.9** (4) **Q.10** (2)
Q.11 (3) **Q.12** (4) **Q.13** (2) **Q.14** (2) **Q.15** (1) **Q.16** (3) **Q.17** (2) **Q.18** (3) **Q.19** (3)